OPERATIONS AND SUPPLY CHAIN MANAGEMENT
Vol. 18, No. 4, 2025, pp. 632–646
ISSN1979–3561 | EISSN 2579–9363

Optimizing Dual-Channel Retail Strategies: The Impact of Return Re-Salability on Inventory and Contractual Decisions

Mohannad Radhi
Department of Industrial Engineering, King Abdulaziz University, Saudi Arabia
Email: mhradhi@kau.edu.sa (*Corresponding Author*)

ABSTRACT

This research investigates a dual-channel retailer (DCR) operating both a physical store and an online store, where customers can return unwanted products for full refunds. By incorporating the re-salability of returned items into the analysis, this study expands the existing literature on dual-channel retailing. Focusing on the retailer's strategy of delegating online store operations to a third-party logistics and service provider, we re-explore three contractual strategies: transaction-based fee, fixed-based fee, and gain-sharing. For each strategy, we determine the optimal inventory levels and, where applicable, the optimal fixed-fee. The primary purpose of this study is to compare decision-making outcomes with and without return re-salability, and to analyze how these decisions shift when re-salability is considered, using a combination of analytical and numerical methods. Through analytical analysis, it is found that when the retailer faces high logistical or purchasing costs, an online store may order more in a re-salable system than in a non-re-salable one. This underscores the importance of not assuming that order quantities are always lower in re-salable systems.

Keywords: *customer returns, dual-channel retailing, newsvendor model, outsourcing, re-salability, service provider, third-party logistics*

1. INTRODUCTION

Zhang *et al.* (2010) claimed that about 80% of all American businesses, including some of the most significant ones in the country, use a dual-channel retail system. Thus, companies can sell their products to customers through both physical and online stores and consequently gain several advantages (Ryan *et al.*, 2013). Wal-Mart, Costco, Barnes and Noble, Nike, Jo-Ann Fabric and Craft Stores, Target, Kohl's, and several others are examples of DCRs and sometimes referred to as click-and-mortar companies. The emergence of pandemic such as COVID-19, the expansion of the internet and the existence of competent third-party logistics providers have made it possible for more retailers to establish a presence online in the last decade. According to Prasertwit *et al.* (2024), e-commerce generated estimate sales of \$5.2 trillion globally in 2021 and is expected to contribute \$8.1 trillion by 2026. This has been an excellent way to reach customers in places where otherwise they would not be possible. Providers have demonstrated great strides in their ability to support services such as transportation, warehousing, inventory management, fleet management and production management. The management of a retailer-provider relationship is a complex task, and there are mixed opinions on the financial effectiveness of outsourcing (Hartmann & de Grahl, 2012). However, when enterprises outsource services, they can improve their competitiveness and overall performances.

A lot of shops in North America offer full refunds for all their customers. Certain retail chains implement a no-questions-asked policy to ensure customers feel confident in their purchases and have no concerns regarding returns. Businesses that adopt such a policy will enjoy a boost in sales, customer satisfaction and loyalty. This also may help in adhering to the commercial laws of a country. On the other hand, it encourages customers to return their purchases more often. Studies have shown that return rates tend to increase as online channels become more available. Indeed, the credibility and suitability of items from online stores are significantly lower compared to those from traditional stores, primarily due to differences in customer interaction and exposure. To reassure buyers, many businesses offer full refunds with their products to make sure they're of good quality. Consequently, an item is likely to yield a higher return rate if it is purchased through the retailer's website rather than in person. As an example, the rate of return on clothing can be as low as 35% when purchased in person but can increase to up to 75% if purchased through the retailer's website (Akçay *et al.*, 2013; Mostard & Teunter, 2006; Vlachos & Dekker, 2003).

Returns have a huge impact on DCRs and one very impactful aspect is operations management. Many new retailers have registered low performances in their first online attempts due the failure in satisfying shoppers' orders and the costs associated with returns. Indeed, unhappy shoppers will exhaust the retailers with both forward and reverse logistical activities and cost. Online stores are often dissuaded from prioritizing customers' orders and returns in favor of more conventional marketing activities. This is understandable; however, an online store will not last long if it does not put enough focus on delivering goods and handling returns. As such, it is critical important for businesses to ensure that the inventory levels of channels are well controlled when considering customer returns.

Currently, various inventory, logistics, and warehousing strategies are commonly employed by DCRs. For instance, some companies consolidate their merchandise in a single warehouse to fulfill the demands of both online and physical outlets. Notable examples of such businesses include The Home Depot and Wal-Mart Stores (Yan, 2008). Other DCRs use two inventory locations or piles, so that customers' orders from each channel are isolated and handled separately. According to Yao *et al*. (2009), Penny tries to avoid inventory mix ups by using this strategy. Other companies do the same to avoid coordination problems between channels and its associate expenses.

Today, it has become more common for online stores to outsource logistics and warehousing tasks. Outsourcing can be a wise business strategy to reinvest time and energy into other aspects of the business while still maintaining high levels of quality. According to Lei *et al*. (2006), most online stores rely on third-party logistics providers to manage their transportation needs. Outsourcing policies range from the commonly used per-service charge model to the less frequent but more collaborative approach of a true partnership involving gain sharing (Min, 2013). The alliance between AutoZone, Inc and Transplace, as well as the alliance between Sheetz Corporation and various logistics providers are recent examples of such partnerships. According to Lei *et al*. (2006), gain sharing is a key piece of the compensation policy that allowed previous partnerships to succeed. Toys“R” Us had a notable partnership with Amazon.com, where Amazon handled order fulfillment, customer service, inventory management, and site development (Berger *et al*., 2006). In this collaboration, Amazon also stored Toys 'R' Us inventory at its distribution centers.

This paper performs inventory management while considering fully reimbursable and re-salable customer returns under three outsourcing agreements, as outlined below:

1. *Transaction-based fee agreement*: Under this agreement, the DCR, herein called the retailer, meets the demand of the physical store and for the online store using separate inventory piles or locations. For managing online orders and returns, the retailer can either use its own fleet for logistics or outsource these tasks to a third-party logistics and service provider, herein called the provider, with compensation based on a per-service agreement. This arrangement does not require a substantial upfront payment to the provider. Since no long-term agreement is established with the retailers, the fee is treated as an exogenous factor. Consequently, a single decision-maker oversees the implementation of this strategy. According to Goertler *et al*. (2024), transaction cost economics, under which this setting falls, has been ranked as the second most frequently used theory in logistic and supply chain management over the past 15 years.
2. *Flat-based fee agreement*: In this policy, the retailer meets the physical store’s demand, while a provider is hired to manage the online store’s demand. The provider, as a Stackelberg leader, charges the retailer a flat fee that is quantity dependent at a predetermine time within the selling season. The retailer, as a Stackelberg follower, chooses the quantity levels for both channels. The inventory decision related to the online store is the consideration of the retailer. Thus, this strategy is similar to that of the vendor-managed inventory (VMI). Goertler *et al*. (2024) also indicated that the application of game theory in logistic and supply chain management is among the most commonly used theories.
3. *Gain-sharing agreement*: Similar to the earlier agreement, the retailer meets the physical store’s demand, while the hired provider meets the online store’s demand. In addition to the retailer's contribution of the seasonal fee, the partners share the profits generated by the online store.

This study makes three key contributions: first, it explores the management of an online store through various outsourcing strategies, taking into account the re-salability of customer returns within a dual-channel retail system; second, it develops mathematical models to optimize order quantities for each strategy under uncertain demand; and third, it compares the insights gained from this study with those from an earlier research that did not account for return re-salability.

This work is organized as follows. In Section 2, we examine current literature related to our study. We introduce our problem and investigate the transaction-based fee agreement in Sections 3 and 4, respectively. Then, we examine the flat-based fee and gain sharing agreements in Sections 5 and 6, respectively. In Section 7, we present comparative and numerical analysis to extract several valuable managerial insights. Finally, section 8 offers conclusions and suggests future research topics.

2. LITERATURE REVIEW

This section examines three areas of literature. It begins with a review of outsourcing third-party logistics and service providers, along with their associated contractual agreements. Next, it explores customer returns in non-competitive settings. Finally, it revisits customer returns within the context of theoretical game frameworks.

Logistics outsourcing is becoming an increasingly popular research area in recent years. Similar to this study, Radhi (2024) examined optimal pricing policies under transaction-based fee, flat-based fee, and gain-sharing agreements when two forms of customer-returns exist. Fallahi *et al*. (2023) analyzed a centralized alliance between a retailer and a third-party logistics provider that incorporated profit-sharing agreement. Their model optimized flow and selling prices, contributing to reduced pollution and harmful emissions. Cao *et al*. (2023) investigated optimal operational and logistical policies for venders utilizing online selling platforms. The study identified conditions under which vendors should operate as suppliers or co-optors and decide between platform logistics or non-platform logistics. Chen *et al*. (2022) explored coordination within a three-echelon closed-loop supply chain involving a manufacturer, retailer, and third-party logistics provider, where the retailer acts as the Stackelberg leader. Tu *et al*. (2022) examined merchants selling agricultural products using third-party logistics providers and online platform. They argued that demand is influenced by both promotional activities and logistical efforts. To align the supply chain, they proposed three types of contracts: fixed-price, revenue-sharing, and cost-sharing. Zhang *et al*. (2021) developed game-theoretical framework for a supply chain comprising of a product supplier, a platform service provider, and a logistics provider. Wang *et al*. (2021) introduced and applied an "altruistic preference joint fixed-cost" contract to coordinate the interactions among a third-party logistics provider, a manufacturer, and an online retailer operating in a cross-shopping retail environment. Giri & Sarker (2017) examined the supply chain’s performance under a system containing manufacturers, a third-party logistics provider, and independent retailers. The demand was both price sensitive and uncertain. Buyback and revenue sharing contracts were utilized to manage the competitive supply chain wherein disruption in production may exist in the manufacturing system. He *et al.* (2016) examined various strategies that can be applied between the manufacturer's and retailer's online stores. They assumed demand to be service level, advertising effort, and price dependent. Jiang *et al.* (2014) examined decision and coordination mechanisms between a manufacturer, a third-party logistics provider, and two competing retailers when the functions of product distribution are used. Cai *et al.* (2013) studied a supplier offering perishable products to a distributor in which both quantity and quality deteriorate throughout the transportation process. Market demand is assumed to be uncertain, as well as price and product freshness dependent. The studied system also exploits a third-party logistics provider for shipping products between the supplier and distributor. Liu *et al.* (2013) examined the fairest revenue-sharing coefficient for a two-echelon system consisting of a provider and a logistics integrator. They also examined the coefficient for a three-echelon system consisting of provider, logistics integrator, and subcontractor. The authors have implemented a profit distribution equity and a fair entropy function in their study. Lei *et al.* (2006) examined the impact of coordination and pricing policies on system's performance given a logistics provider's cost function that is naturally concave. Jharkharia & Shankar (2007) suggested two screening methods that can help in selecting a third-party logistics provider, namely, analytic network process-based final selection and preliminary screening. Fabbe-Costes *et al.* (2009) looked at how logistics providers could support supply chain integration and customers' performance. Lim (2000) established a game-theoretic model that investigates the contractual policies between third-party logistics buyer and provider. The study assumes that service quality and service cost parameters are unknown to the buyer. Despite the significance of resalable returns on retailing systems, none of the above papers have studied their impact on contractual policies that may form between a retailer and a provider when the inventory policy is a vital decision for the retailer.

Furthermore, various scholars have analyzed customer returns within centralized or cooperative frameworks, often considering a single-echelon system in this context. Nageswaran *et al.* (2020) studied pricing and return policies for a dual-channel retailer managing both physical and online stores. They assumed full refunds for returns made to the physical store, while considered full and partial refunds for returns made to the online store. Reimann (2016) examined a system where refurbished returns can be utilized to meet demands that is greater than the order quantity. Akçay *et al.* (2013) examined a retailing system where customers can distinguish between a new item and a returned item. Yu & Goh (2012) evaluated eight different returns scenarios that a seller may encounter. Those scenarios considered the return timing and its significance, penalty, and recoverability. Similarly, Vlachos & Dekker (2003) evaluated six different returns scenarios. The scenarios considered the return re-salability in the primary market, and recoverability. Mostard *et al.* (2005), and Mostard & Teunter (2006) studied a system that allows an item to be repetitively resold. Similarly, Wang *et al.* (2010) divided the selling season into three sub-periods depending on the consumption of demand for new and returned products.

In addition, other scholars have studied customer returns within decentralized or competitive frameworks, often considering a two-echelon system in this context. Radhi (2022) explored service competition between a dual-channel retailer and a manufacturer, focusing on how service levels influence channel demand and preferred channel of return. Zhang *et al.* (2021) analyzed the pricing strategies of a risk-averse dual-channel retailer and a risk-neutral manufacturer. They assumed full refunds for returns to the physical store, while online returns were considered eligible for either full or partial refunds. Their study examined how the retailer's risk aversion and the consumer return rate influence the overall performance of the retail system. Jin *et al.* (2020) explored the competitiveness and effectiveness of cross-channel return policies when two DCRs compete. The study assumed that items returned to the physical store generate greater salvage value than items returned to the online store. Liu *et al.* (2020) investigated pricing strategies involving a retailer with a physical store and a manufacturer with an online store. They identified the conditions under which single or dual money-back guarantee return policies would be optimal. Radhi & Zhang (2018) have examined pricing policies for a centralized and decentralized dual-channel retailer while considering both forms of returns, namely, same- and cross-channel returns. They have also examined inventory policies under four different return schemes and similar return settings (Radhi & Zhang, 2019). Similar to this study, both papers assumed that resalable returns can be used one more time to satisfy demand within the same selling season. Chen & Bell (2012) analyzed a retail system featuring two types of shoppers: return-sensitive shoppers, who pay more and benefit from a more flexible return policy, and price-sensitive shoppers, who pay less and face a stricter return policy. Also, Chen & Grewal (2013) considered rivalry between a well-founded retailer that applies money back guarantee policy and a new market retailer. Similarly, Chen & Zhang (2011) used two game-theoretical schemes that examine competition between two retailers with money back guarantee policy. Balakrishnan *et al.* (2014) examined the browsing and switching behavior utilized by exploiting customers. They studied what effect such a behavior has on stores' profits and prices when returns are possible for online purchases only. Ofek *et al.* (2011) considered competition in pricing and customer service level between two retailers running a single channel or dual channels. One may notice that the previous papers studied the negative effect of customer returns on the different tiers of a supply chain. However, that was done without considering the provider's role in the retailing process.

Similar to Radhi (2018), this study looks at a dual-channel retailer facing customer returns and the option to outsource the online store's management to a third-party logistics and service provider. Hence, we investigate the retailer's inventory and the provider's fee decisions under three contractual agreements: transaction-based fees, flat-based fees, and gain-sharing. However, a key contribution here is the consideration of return re-salability and its impact on optimal decisions and system's performance.

3. PROBLEM STATEMENT

This research studies a dual-channel retailer where both the physical and online stores offer the same perishable product to shoppers. Thus, newsvendor modeling technique is used where total demand is assumed to be stochastic. A portion of customers prefers to shop through the physical store, while the rest prefers to shop through the online store. Since shoppers prefer one store over another, their unmet demand from that store is lost. The previous assumption is reasonable for shoppers that are store loyal. For example, online store's customers enjoy the process of shopping without time or location restrictions. They are more likely to shop through another online store if the needed item is not available. On the contrary, physical store's customers enjoy touching and feeling the product before making buying decisions. Thus, they are more likely to shop through another physical store if the needed item is not available. Due to the shortage of the selling season, this paper assumes no transshipment between stores.

In accordance with the previously explained customer preference, we assume that observed total sales x is drawn from a probability distribution and splits between the two stores. Therefore, the physical store's observed sales and the online store's observed sales are θx and $(1-\theta)x = \bar{\theta}x$, respectively. Splitting sales between channels has been used in many studies pertaining to the topic of dual-channel supply chain (Chiang & Monahan, 2005; Yao *et al.*, 2009). Due to tractability and boundness, we presume that the total sales x is Uniformly distributed, i.e., $x \sim U[a, b]$, where $x = a$ represents the minimum potential total sales and $x = b$ represents the maximum potential total sales. The chosen distribution has enough embedded generality to capture variability within a selling season.

Shoppers can return their purchased items and acquire full refunds under the condition that their returns are as good as new, and they are returned within a time-frame specified by the retailer. Consequently, a ratio r_r from the physical store's total sales is returned to the store by shoppers, and a ratio r_o from the online store's total sales is returned to the store through the available logistical services. The ratio assumptions for returns have been used in several existing works (Chen & Grewal, 2013; Mostard *et al.*, 2005; Mostard & Teunter, 2006; Radhi, 2022, 2024; Radhi & Zhang, 2018, 2019; Vlachos & Dekker, 2003). In addition, literature has used several assumptions and modeling techniques to simplify the complexity coupled with return re-salability. For example, several papers assumed that returns can be re-soled infinitely in a selling season (Mostard *et al.*, 2005; Mostard & Teunter, 2006), while others assumed they can be resold only once in a selling season (Radhi & Zhang, 2018, 2019; Vlachos & Dekker, 2003). Due to the advancement in technology and the demand exerted by customers to always have new designs and products, selling seasons are shortening. Therefore, the second assumption is used in this study along with the fact that all returns are as good as new. Hence, the order quantity assigned for the online store, i.e. Q_o, can satisfy a maximum online store's total sales of $Q_o(1 + r_o) = \lambda Q_o$. Also, the order quantity assigned for the physical store, i.e. Q_r, can satisfy a maximum physical store's total sales of $Q_r(1 + r_r) = \delta Q_r$.

We assume that the selling price p is exogenously determined. Also, an item that never been sold or an item that never made it as a final sale, i.e. returned for the second time, is salved for a value of s. Such an assumption is realistic, as several enterprises accept returns only in their original form. The salvage value is assumed to be less than the unit's purchasing cost, i.e. $w > s$, otherwise the profit function will be unbounded above. Due to economy of scale, the retailer acquires a higher logistical fee per transaction compared to the provider, i.e. $h_r > h_l$.

Return re-salability plays a critical role in inventory management and contractual agreements as it directly impacts the efficiency and profitability of supply chain operations. Hence, considering re-salability while building up inventory minimizes waste, reduces inventory holding costs, and enhances the overall value recovery process. Conversely, if not considered, businesses risk significant financial losses due to unsellable inventory, increased disposal costs, and wasted resources. This oversight can also lead to inefficiencies in reverse logistics and strain contractual relationships. Therefore, by considering returns re-salability, this work expands the inventory models proposed by Radhi (2018) for a dual-channel retailer partnering with a third-party logistics and service provider. Also, Table 1 below summarizes both parameters and decision variables (DVs), as indicated within the table, used in developing the mathematical models.

Table 1 Notations.

Notation	**Description**
r_r	Rate at which an item obtained from the traditional store is returned to the store
r_o	Rate at which an item obtained from the online store is returned to the store
w	Per unit purchasing cost
p	Unit selling price
s	Salvage value where $w > s$
h_r and h_l	Per unit forward and reverse logistical cost acquired by the retailer and provider, respectively where $h_r > h_l$
g	Shortage cost
x	Total sales
$f(x)$	Probability density function for total sales
θ	Customer preference for the offline store
$\bar{\theta} = 1 - \theta$	Customer preference for the online store
Q_r and Q_o **(DVs)**	Quantities ordered for offline and online stores, respectively
F **(DV)**	Seasonal fee charged by the provider for managing each online item. This variable will be further discussed in Section 5.

4. TRANSACTION-FEE STRATEGY ($i = 1$)

In this strategy, the provider is not a significant player, and the retailer entirely manages the decisions of the retailing system. Thus, there is no strategic partnership between the two entities. Sales for online and physical stores are satisfied from separated inventories within the retailing facility or from two separate locations. Giant retailers, such as Penny, use inventory separation to assure commenting to online store's sales. Online orders or returns are transported from or back to the online store through the retailer's fleet or the provider's per transaction payment services. Unlike Radhi (2018), this work will not consider the cross-channel return option to maintain model tractability and simplicity while considering re-salability. Accordingly, the retailer maximizes two profit functions by selecting the following decision variables: the inventory level Q_r for the physical store and Q_o for the online store.

Similar to Radhi (2024) and Radhi & Zhang (2018, 2019), the revenue generated by fulfilling a single sale from the online store is $p_1^o = (1 - r_o)p - \lambda h_r + \frac{r_o^2}{\lambda}s$. The term $(1 - r_o)$ represents the ratio of final sales and contributes positively due to the selling price. The term λ represents the ratio of delivery and return and contributes negatively due to the shipping expenses. The term $\frac{r_o^2}{\lambda}$ represents the ratio of second time return and contributes positively due to end of selling season salvaging. Similarly, the revenue generated by fulfilling a single sale from the physical store is $p_i^r = (1 - r_r)p + \frac{r_r^2}{\delta}s$. Due to the similarity, a detailed description of the earlier relationship has been excluded. One may observe that the revenue generated from fulfilling a single sale with return re-salability is lower compared to the case without re-salability. Specifically, respective reductions for online and physical stores are $\frac{r_o s}{\lambda}$ and $\frac{r_r s}{\delta}$. However, this does not imply that the latter scenario is more profitable. Subsequent analysis will demonstrate that re-salability leads to lower quantities, allowing for greater profitability improvements.

Due to return re-salability, the quantity Q_o acquired by the online store can satisfy sales equal to $Q_o(1 + r_o) = \lambda Q_o$. Since each sold item generates a revenue of p_1^o, then $\lambda Q_o p_1^o \geq w Q_o$. That is the total acquired revenue must be greater than or equal to the total purchasing cost. Thus, $\lambda p_1^o \geq w \geq s$ indicating that $p_1^o \gg \frac{w}{\lambda} \gg \frac{s}{\lambda}$. Otherwise, the online store will not be profitable. Similarly, the total acquired revenue acquired by the physical store, i.e. $(1 + r_r)Q_r p_i^r = \delta Q_r p_i^r$, should be greater that or equal to the total purchasing cost, i.e. wQ_r. Thus, the following relationship should hold: $p_i^r \gg \frac{w}{\delta} \gg \frac{s}{\delta}$. Otherwise, the physical store will not be profitable.

In addition, the total expected profit for the physical store is presented in Equation 1.

$$\pi_i^r(Q_r) = \int_a^{\frac{\delta Q_r}{\theta}} \left(\theta x p_i^r + \left(Q_r - \frac{\theta}{\delta}x\right)s\right) f(x)dx + \int_{\frac{\delta Q_r}{\theta}}^b (\delta Q_r p_i^r - (\theta x - \delta Q_r)g) f(x)dx - wQ_r \tag{1}$$

The first term in Equation 1 represents the expected profit when the store's sales θx is less than or equal to the sales upper limit δQ_r. The term consists off the expected revenue acquired by the store and the salvage value for items that have never been sold. Note that the expected salvage value for non-resalable returns is accounted for in the expected revenue, i.e. p_i^r. The second term represents the expected profit when the store's sale θx is more than or equal the sales upper limit δQ_r. The term consists off the expected revenue acquired by satisfying δQ_r orders excluding the shortage cost for unsatisfied sales. The third term is the cost of purchasing the inventory level Q_r. In a similar fashion, the total expected profit for the online store is depicted in Equation 2.

$$\pi_1^o(Q_o) = \int_a^{\frac{\lambda Q_o}{\bar{\theta}}} \left(\bar{\theta} x p_1^o + \left(Q_o - \frac{\bar{\theta}}{\lambda}x\right)s\right) f(x)dx + \int_{\frac{\lambda Q_o}{\bar{\theta}}}^b (\lambda Q_o p_1^o - (\bar{\theta} x - \lambda Q_o)g) f(x)dx - wQ_o \tag{2}$$

Proposition 1. *The optimal inventory policy for the offline store and the online store are provided respectively as follows:*

$$Q_i^r = \frac{\theta}{\delta}\left(b - \frac{(b-a)(w-s)}{\delta p_i^r + \delta g - s}\right)$$

$$Q_1^o = \frac{\bar{\theta}}{\lambda}\left(b - \frac{(b-a)(w-s)}{\lambda p_1^o + \lambda g - s}\right)$$

Proof of Proposition 1. Equation (1) can be rewritten as the following: $\pi_i^r(Q_r) = \theta\left(p_i^r - \frac{s}{\delta}\right)\frac{a+b}{2} - (w-s)Q_r + \frac{p_i^r + g - \frac{s}{\delta}}{2(b-a)}\left(2\delta b Q_r - \frac{\delta^2 Q_r^2}{\theta} - \theta b^2\right)$. Thus, $\frac{\partial^2 \pi_i^r}{\partial Q_r^2} = -\frac{\delta^2\left(p_i^r + g - \frac{s}{\delta}\right)}{\theta(b-a)}$. Since $p_i^r \gg \frac{s}{\delta}$, then $\frac{\partial^2 \pi_i^r}{\partial Q_r^2} < 0$ and the store's profit is strictly concave in Q_r. By solving $\frac{\partial \pi_i^r}{\partial Q_r} = 0$, we can find the optimal value of Q_i^r. Also, Equation (2) can be rewritten as the following: $\pi_1^o(Q_o) = \bar{\theta}\left(p_1^o - \frac{s}{\lambda}\right)\frac{a+b}{2} - (w-s)Q_o + \frac{p_1^o + g - \frac{s}{\lambda}}{2(b-a)}\left(2\lambda b Q_o - \frac{\lambda^2 Q_o^2}{\bar{\theta}} - \bar{\theta} b^2\right)$. Thus, $\frac{\partial^2 \pi_1^o}{\partial Q_o^2} = -\frac{\lambda^2\left(p_1^o + g - \frac{s}{\lambda}\right)}{\bar{\theta}(b-a)}$. Since $p_1^o \gg \frac{s}{\lambda}$, then $\frac{\partial^2 \pi_1^o}{\partial Q_o^2} < 0$ and the store's profit is strictly concave in Q_o. By solving $\frac{\partial \pi_1^o}{\partial Q_o} = 0$, we can find the optimal value of Q_1^o. □

Proposition 1 offers closed-form formulas of the physical and online stores' optimal inventory levels. If the totality of customers prefers the physical store (i.e. $\theta = 1$), and the retailer experiences no or does not allow returns (i.e. $r_r = 0$), then the optimal inventory level is equivalent to that of the classical newsvendor problem. Also, if the values of δ and λ are equivalent to one, then the optimal inventory functions will provide similar expressions given in Radhi (2018), i.e. without re-salability considerations. However, this only occurs when the rates of returns are zeros, in such a case re-salability has no meaning. In addition, the ratio multiplied by a customer preference to a certain channel should be much greater than one, otherwise inventory levels will contain a fraction of an item. That is $\frac{1}{\delta}\left(b - \frac{(b-a)(w-s)}{\delta p_i^r + \delta g - s}\right) \gg 1$ and $\frac{1}{\lambda}\left(b - \frac{(b-a)(w-s)}{\lambda p_1^o + \lambda g - s}\right) \gg 1$. This notion can be used to show that a system with re-salability will optimally contain lower levels compared to a system without re-salability. However, due to complexity, this attempt is omitted. Finally, as customer preference to the physical channel, i.e. θ, increases, the order quantity in the physical store increases and the order quantity in the online store decreases and vice versa. Such adaptation of quantities to the change in customer preference is intuitive and well documented in the dual/omni-channel supply chain literature. Similar findings were achieved in all the upcoming strategies. Ended, another outsourcing policy that demands a higher commitment from the provider side is considered next.

5. FIXED-FEE STRATEGY ($i = 2$)

Currently, numerous retailers outsource online store's orders fulfillment and customer returns to a provider for improvement in efficiency, reduction in cost, and enhancement in customer satisfaction. The related processes either take place in a retailer-owned warehouse (e.g., the contractual agreement between the retailer-HP and the provider-FedEx) or in a provider-owned warehouse (e.g., the contractual agreement between the retailer-Global Stores and the provider-Kmart.com) (Radhi, 2018; Yao *et al.*, 2009). Amazon's Fulfillment Center (FBA) provides a more sophisticated and developed example of the later strategy.

In this model, the retailer still retains control over the online store's inventory strategy. Thus, compensating the provider with a seasonal fee tied to sales (Q_o), effectively linking the scale of the retail operation (effort) to the fee (gain). However, with re-salability, the quantity ordered is lower compared to the case without re-salability, while still achieving the same level of sales. This might create an opportunity for the retailer to hide some expenses and expand its profit. Thus, it is vital to evaluate the optimal responses in both cases and determine how re-salability impact rivals' decisions and performances.

In this strategy, players engage in a Stackelberg game, making independent decisions to maximize their individual profits. The provider is regarded as the leader in the game, while the retailer assumes the role of the follower. Similar to Radhi (2018), the decisions sequence is depicted below:

- The provider maximizes the expected profit using the online store's optimal response function, given the seasonal fee (F) charged for managing each online item. Consequently, the optimal fee is calculated under the assumption that the provider has complete knowledge of the parameters related to online store's sales and return behavior.
- Responding to the decision taken by the provider, the retailer sets the optimal inventory level for the online store (i.e., Q_o) with the goal of maximizing the expected profit.

Similarly, Radhi (2024) assume that the retailer determines the pricing policy in response to the provider's seasonal fee, while Giri & Sarker (2017) assume that the retailer determines the order quantity in response to the provider's service charge. The outcome of this strategy does not influence the decision-making process within the physical store. In addition, since handling sales and returns is the provider's responsibility, the revenue generated from fulfilling a single online sale changes to $p_2^o = (1 - r_o)p + \frac{r_o^2}{\lambda}s = p_1^o + \lambda h_r$. Since $p_2^o > p_1^o$ and $h_r > h_l$, then $p_2^o > p_2^o - \lambda h_l > p_1^o \gg \frac{w}{\lambda} \gg \frac{s}{\lambda}$ or $\lambda p_2^o > \lambda p_2^o - \lambda^2 h_l > \lambda p_1^o \gg w \gg s$.

5.1 The Retailer's Problem

The expected profit for the online store can be modeled as follows:

$$\pi_2^o(Q_o|F) = \int_a^{\frac{\lambda Q_o}{\bar{\theta}}} \left(\bar{\theta} x p_2^o + \left(Q_o - \frac{\bar{\theta}}{\lambda} x\right) s\right) f(x)dx + \int_{\frac{\lambda Q_o}{\bar{\theta}}}^{b} (\lambda Q_o p_2^o - (\bar{\theta} x - \lambda Q_o) g) f(x)dx - (w + F)Q_o \qquad (3)$$

Given the similarity, a thorough explanation for the prior profit function is excluded. Next proposition obtains the optimal inventory level for the online store given the seasonal fee F.

Proposition 2. *The optimal inventory level for the online store is given below:*

$$Q_2^o = \frac{\bar{\theta}}{\lambda}\left(b - \frac{(b - a)(w + F - s)}{\lambda p_2^o + \lambda g - s}\right)$$

Proof of Proposition 2. For the decision of the online store, Equation (3) can be reworked as the following: $\pi_2^o(Q_o|F) = \bar{\theta}\left(p_2^o - \frac{s}{\lambda}\right)\frac{a+b}{2} - (w + F - s)Q_o + \frac{\left(p_2^o + g - \frac{s}{\lambda}\right)}{2(b-a)}\left(2\lambda b Q_o - \frac{\lambda^2 Q_o^2}{\bar{\theta}} - \bar{\theta} b^2\right)$. Given F, $\frac{\partial^2 \pi_2^o}{\partial Q_o^2} = -\frac{\lambda^2\left(p_2^o + g - \frac{s}{\lambda}\right)}{\bar{\theta}(b-a)} < 0$. Thus, the function is strictly concave in Q_o. By solving $\frac{\partial \pi_2^o}{\partial Q_o} = 0$, the online store's optimal order quantity (Q_2^o) can be found. □

Similar to Proposition 1, Proposition 2 demonstrates that when λ equals one, the optimal inventory function results in an expression similar to that in Radhi (2018), i.e. re-salability is not taken into account. This, however, only holds true when the return rate is zero, as in this scenario, re-salability becomes meaningless. By comparing the previous two propositions, one may find that $Q_1^o < Q_2^o$ under the condition $h_r > \frac{F\left(\lambda(1 - r_o)p + r_o^2 s + \lambda g - s\right)}{\lambda^2 (w + F - s)}$. That is, if the retailer's logistical performance is lacking (i.e., if h_r exceeds a specific threshold), then partnering with a provider allows for a higher online store's inventory level. Hence, inventory control is significantly dependent on the type of compensation policy used, and the level of efficiency or economy of scale that can be attained by the retailer.

5.2 The Provider's Problem

The provider's profit function can be formulated as follows:

$$\pi_2^L(F) = Q_o F - \int_a^{\frac{\lambda Q_o}{\bar{\theta}}} \bar{\theta} x \lambda h_l f(x)dx - \int_{\frac{\lambda Q_o}{\bar{\theta}}}^{b} Q_o \lambda h_l f(x)dx \qquad (4)$$

The first term in Equation (4) represents the total seasonal fee collected by the provider for offering logistical and handling services to the retailer. The second term denotes the expected handling costs incurred by the provider when the online store's sales, $\bar{\theta} x$, remain within or below the upper sales limit, λQ_o. The third term, on the other hand, accounts for the expected handling costs when the online store's sales exceed the upper sales limit. It is worth noting that the expression λh_l represents the expected handling cost per unit of customer sales handled by the logistical provider. Following this, we proceed to calculate the provider's optimal seasonal fee by utilizing the retailer's best response function.

Proposition 3. *The optimal seasonal fee for each product handled by the provider is:*

$$F_2 = \frac{\frac{b}{(b-a)}\left(p_2^o + g - \frac{s}{\lambda} + h_l r_o\right)(\lambda p_2^o + \lambda g - s) - (w-s)\left(p_2^o + g - \frac{s}{\lambda} - h_l(1-r_0)\right)}{2\left(p_2^o + g - \frac{s}{\lambda} - \frac{h_l}{2}(1-r_o)\right)}$$

Proof of Proposition 3. Equation (4) can be presented as $\pi_2^L(F) = Q_oF - \bar{\theta}\lambda h_l \frac{a+b}{2} - \frac{\lambda h_l}{2(b-a)}\left(2Q_ob - b^2\bar{\theta} - \frac{\lambda(2-\lambda)}{\bar{\theta}}Q_o^2\right)$. Substitute the expression Q_2^o as a function of F into $\pi_2^L(F)$. Thus, $\frac{\partial^2\pi_2^L}{\partial F^2} = \frac{-2(b-a)\bar{\theta}}{\left(p_2^o+g-\frac{s}{\lambda}\right)^2}\left(p_2^o + g - \frac{s}{\lambda} - \frac{h_l}{2}(1-r_o)\right)$. Since $p_2^o - \lambda h_l \gg \frac{s}{\lambda}$, then $p_2^o \gg \frac{s}{\lambda} + \lambda h_l$. Therefore, $\frac{\partial^2\pi_2^L}{\partial F^2} < 0$ and the function is strictly concave in F. By solving $\frac{\partial \pi_2^L}{\partial F} = 0$, the optimal seasonal fee F_2 can be found. □

Proposition 3 provides a closed form expression for the optimal seasonal fee earned for each online product managed by the provider. One may notice that this fee is not influenced by customer preference to a certain channel. Once again, the inexistence of online returns provides similar seasonal fee calculations compared to the case without re-salability. However, this research is focused on returns and the consequence possibility of re-salability. Thus, several vital questions must be addressed. For instance, will re-salability lead to a significant shift in the provider's position? Will the provider exert greater or lesser pressure through the imposed fee policy?

In addition, using F_2 expression, one may find an alternative expression of Q_2^o. That is $Q_2^o = \frac{\bar{\theta}}{2\lambda}\left(\frac{b(\lambda p_2^o+\lambda g-w-\lambda h_l)+a(w-s)}{\lambda p_2^o+\lambda g-s-\lambda h_l\frac{1-r_o}{2}}\right)$. If, as stated before, $\lambda p_2^o - \lambda^2 h_l \gg w \gg s$, then $\lambda p_2^o \gg w + \lambda^2 h_l \gg s + \lambda^2 h_l$. Therefore, the positivity of Q_2^o is assured. In another word, the retailer will always cooperate with the provider even if this policy provides lower performance compared to the transaction-fee policy. Hence, it is important for the retailer to compare between the losses caused by double marginalization and those caused by high handling fees. Finally, one can easily derive the rate at which online order quantity decreases with increasing customer preference for physical stores

6. GAIN-SHARING STRATEGY
$(i = 3)$

Another approach that is used to navigating the challenges of inventory management within a dual-channel retailing system is the adoption of a gain-sharing strategy. In this model, the provider is compensated with a fixed-fee for each managed online item and a share of the gains generated by the channel. This mutually beneficial arrangement aligns the incentives of the retailer and the provider, encouraging them to work collaboratively to optimize the performance of the online channel and the overall supply chain (Hartmann & de Grahl, 2012). Hence, Kim & Lee (2024) highlighted that firms involved in collaborative partnership tend to achieve higher performance levels compared to those that are not. According to Keränen *et al.* (2023), retailers prefer gain-sharing contracts with a fixed-fee over those without a fixed-fee. This concept is widely used in the context of employee compensation schemes, where workers receive a salary and a pay-for-performance fee that links their financial rewards to their performance.

The partnership between Toys "R" Us and Amazon.com, established in 2001, serves as an illustrative example. Toys "R" Us was responsible for sourcing, purchasing, and managing the inventory, while Amazon.com handled order fulfillment, customer service, and maintained inventory in its own distribution facilities. The agreement stipulated that Amazon.com would be remunerated through per-unit payments, fixed payments, and a share of the total gain.

The impact of product returns and their subsequent re-salability on gain-sharing alliances is an important consideration. High return rates can decrease the quantity needed to fulfill sales due to re-salability. This would lead to a reduced quantity-based flat fee while increasing the cost of handling returns. Such changes could directly affect the flow of compensation, putting the alliance at risk. This could create tension, as the provider may feel inadequately compensated for the increased reverse logistics burden. Conversely, efficiently managing returns and accounting for re-salability can boost profitability, leading to greater gains for both parties. Hence, a clear agreement on how to handle re-salable returns and incorporate their effect into the gain-sharing calculation is crucial for a successful and sustainable partnership.

In addition, due to the significant risk associated with the gain sharing strategy, it is rarely being used despite the possible improvements in a company's performance, productivity and profitability (Min, 2013). Another reason that discouraged the use of gain sharing policy is the hardship that comes with specifying partners' share such that a win-win situation is achieved (Liinamaa *et al.*, 2016). The stress applied on the provider to achieve predetermined outcomes may leave the management unwilling to accept such a policy.

Here, we assume that the partners share all rates, cost, and revenue related parameters. Additionally, the revenue from the online store is divided between the partners based on their respective market positions, negotiation power, and assigned responsibilities. Consequently, the retailer obtains a share $\emptyset$ ranging from 0 to 1, while the provider receives the remaining share, i.e. $1 - \emptyset$. Furthermore, the provider is granted a sales-dependent, fixed seasonal fee on top of their share. Keränen *et al.* (2023) adopted a similar assumption without considering the varying market power of the different players, which could significantly impact the preferred outcome.

The physical store's order quantity remains the same as in the previous two sections. However, the revenue generated from a single online sale is $p_3^o = (1 - r_o)p + \frac{r_o^2}{\lambda}s - \lambda h_l = p_2^o - \lambda h_l$. It is intuitive to see that $p_2^o > p_3^o > p_1^o \gg \frac{w}{\lambda} \gg \frac{s}{\lambda}$ or $\lambda p_2^o > \lambda p_3^o > \lambda p_1^o \gg w \gg s$. Again, we utilize Stackelberg competition scheme with the same distribution of power. While the retailer's profit function from the online store is $\pi_3^o(Q_o|F) = \emptyset\pi_{gs} - FQ_o$, the provider's profit function is $\pi_3^L(F) = (1 - \emptyset)\pi_{gs} + FQ_o$. Notice that π_{gs} represents the total revenue to be shared among partners.

$$\pi_{gs} = \int_{a}^{\frac{\lambda Q_o}{\bar{\theta}}} \left(\bar{\theta} x p_3^o + \left(Q_o - \frac{\bar{\theta}}{\lambda} x\right) s\right) f(x)dx + \int_{\frac{\lambda Q_o}{\bar{\theta}}}^{b} (\lambda Q_o p_3^o - (\bar{\theta} x - \lambda Q_o) g) f(x)dx - wQ_o \tag{5}$$

Proposition 4. *The optimal inventory level for the online store and the optimal fixed-fee for each online product handled by the provider are depicted, respectively, as:*

$$Q_3^o = \frac{\bar{\theta}}{\lambda}\left(b - \frac{(b-a)\left(\frac{F}{\emptyset} + w - s\right)}{\lambda\left(p_3^o + g - \frac{s}{\lambda}\right)}\right)$$

$$F_3 = \frac{\emptyset^2}{1+\emptyset}\left(\frac{b\lambda\left(p_3^o + g - \frac{w}{\lambda}\right) + a(w-s)}{(b-a)}\right) \geq 0$$

Proof of Proposition 4. The total revenue function (i.e., Equation (5)) can be rewritten as: $\pi_{gs} = \bar{\theta}\left(p_3^o - \frac{s}{\lambda}\right)\frac{a+b}{2} - (w-s)Q_o + \frac{\left(p_3^o + g - \frac{s}{\lambda}\right)}{2(b-a)}\left(2\lambda Q_o b - \frac{\lambda^2 Q_o^2}{\bar{\theta}} - \bar{\theta} b^2\right)$. Given F, $\frac{\partial^2 \pi_3^o}{\partial Q_o^2} = -\frac{\emptyset \lambda^2 \left(p_3^o + g - \frac{s}{\lambda}\right)}{\bar{\theta}(b-a)} < 0$. Thus, the online store's profit function (i.e., π_3^o) is strictly concave in Q_o. By solving $\frac{\partial \pi_3^o}{\partial Q_o} = 0$, we can find the optimal response function for the online store Q_3^o. Substitute the expression of Q_3^o in the function π_3^L. Since, $\frac{\partial^2 \pi_3^L}{\partial F^2} = -\frac{\bar{\theta}(b-a)(1+\emptyset)}{\lambda^2 \emptyset^2 \left(p_3^o + g - \frac{s}{\lambda}\right)} < 0$, then the function is strictly concave in the fixed-fee, i.e., F. By solving $\frac{\partial \pi_3^L}{\partial F} = 0$, we can find the optimal expression of F_3. □

Proposition 4 studies partners' optimal decisions when the compensation policy involves both fixed-fee and gain-sharing. One can substitute the value of F_3 into Q_3^o to get the following expression $Q_3^o = \frac{\bar{\theta}}{\lambda}\left(\frac{b(\lambda p_3^o + \lambda g - w) + a(w-s)}{(\lambda p_3^o + \lambda g - s)(1+\emptyset)}\right) > 0$. The non-negativity of the order quantity is consistently ensured, which suggests that the store is not compelled to cease operations. Radhi (2018) has also demonstrated that the ordered quantity under this strategy is guaranteed to exceed the quantity under the fixed-fee strategy, i.e. $Q_3^o > Q_2^o$. A system with re-salability exhibits a similar relationship, although it would be considerably more complex to provide evidence for this claim.

Given that $\frac{\partial F_3}{\partial \emptyset}$ is positive, F_3 may achieve its top value when $\emptyset = 1$. When $\emptyset = 1$, $F_2 - F_3$ would ensure a positive outcome, i.e. $F_2 - F_3 = \frac{\frac{\lambda^2 b h_l}{2(b-a)}\left(3\left(p_2^o + g - \frac{s}{\lambda} - h_l\right) + h_l(\lambda+1)\right) + (w-s)\frac{h_l(1-r_o)}{2}}{2\left(p_2^o + g - \frac{s}{\lambda} - h_l\right) + h_l\lambda} > 0$. Hence, strategy 2 results in a higher fixed-fee compared to strategy 3. In other words, the mere idea of sharing gains with the retailer would ease competition and generate a lower initial fee, even if it is not actually implemented. Hence, the more desirable the share received by the provider, the lower the fixed-fee required (i.e. $\frac{\partial F_3}{\partial \emptyset} = \frac{(\emptyset+2)F_3}{\emptyset(\emptyset+1)} > 0$) and the more encouraged the retailer to increase the ordered quantity (i.e. $\frac{\partial Q_3^o}{\partial \emptyset} = -\bar{\theta}\frac{b(\lambda p_3^o + \lambda g - \mathrm{w}) + a(w-s)}{\lambda(\lambda p_3^o + \lambda g - \mathrm{s})(1+\emptyset)^2} < 0$). This will have a positive reflection on the total channel's performances.

However, partners can push their alliance to a new dimension, thereby achieving greater online surplus π_{gs}. To do so, they share the channel's gains without any upfront fixed-fee hurdle in a fully centralized fashion. Consequently, they avoid all forms of double marginalization. In such a full gain-sharing case, the sole online decision will be the order quantity $\widetilde{Q_o^3}$, which can be calculated using Q_3^o when that $F = 0$ and that induce a higher order quantity. Also, the online profitability will be denoted as $\widetilde{\pi_{gs}}$. Notice that, for such an integration strategy to be effective, it must ensure a higher surplus for each partner. Hence, the minimum acceptable share for the retailer is $\max\left\{\frac{\pi_{gs}}{\widetilde{\pi_{gs}}}, \frac{\pi_2^o}{\widetilde{\pi_{gs}}}\right\}$, while the minimum acceptable share for the provider is $1 - \min\left\{\frac{\widetilde{\pi_{gs}} - \pi_2^L}{\widetilde{\pi_{gs}}}, \frac{\widetilde{\pi_{gs}} - \pi_3^L}{\widetilde{\pi_{gs}}}\right\}$.

Table 2 Numerical example ($r_o = 0.3, r_r = 0.15, p = 200, h_r = 20, h_l = 5, w = 40, s = 10, g = 5, \theta = 0.2$).

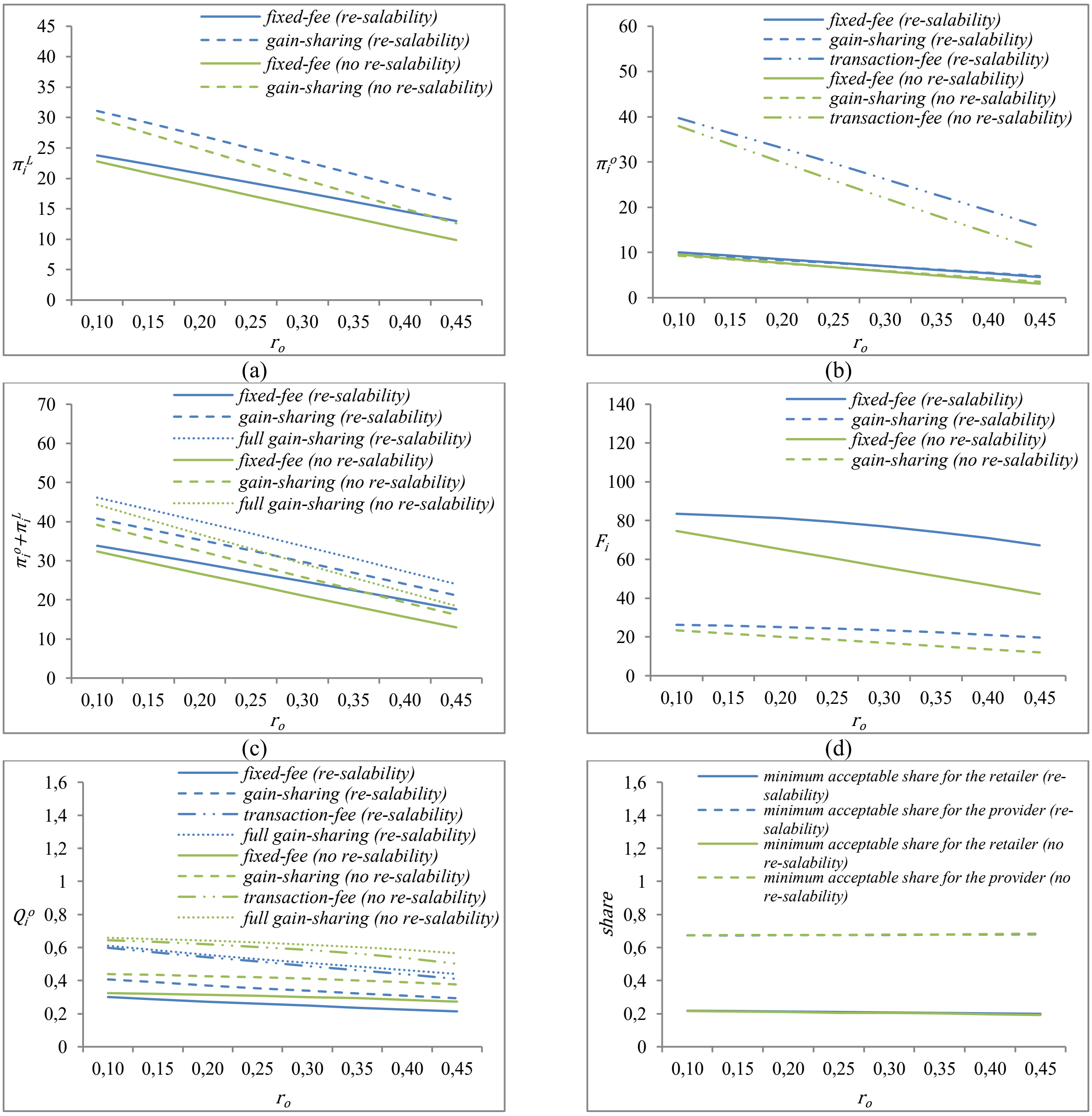
Strategy			**Transaction-Fee Strategy**		**Fixed-Fee Strategy**		**Gain-Sharing Strategy**		**Full Gain-Sharing Strategy**	
Theory Supporting the Calculations			Transaction cost economics theory (sole decision maker with no partnership)		Game theory with Stackelberg setting (low level partnership)		Game theory with Stackelberg setting (medium level partnership)		Centralization (high level partnership)	
Re-Salability (considered or not considered)			✓	✗	✓	✗	✓	✗	✓	✗
Optimal Decisions	Q_i^r	**Optimal Value**	0.15	0.16	0.15	0.16	0.15	0.16	0.15	0.16
		Decision Maker	Retailer		Retailer		Retailer		Retailer	
	Q_i^o	**Optimal Value**	0.49	0.59	0.25	0.30	0.34	0.41	0.51	0.62
		Decision Maker	Retailer		Retailer		Retailer		Centralized	
	F_i	**Optimal Value**	NA		77	56	23.5	16.9	NA	
		Decision Maker			Provider		Provider			
Total Profitability	**Retailer**		37.6	32.7	18.32	16.52	18.29	16.65	18.32 to 22.26	16.65 to 20.14
	Provider		16.5 (if used)	15.3 (if used)	17.7	15.3	22.84	19.89	22.84 to 26.78	19.89 To 23.37

7. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

The aim of this numerical analysis is to provide deeper insights into the profitability and optimal decision-making of a dual-channel retailer managing customer returns, in conjunction with an outsourced third-party logistics provider. The study will evaluate and compare the impact of market conditions on the retailing system, both with and without re-salability. The findings offer valuable managerial insights, particularly in understanding how re-salability influences the decision-making process. To focus on decision behavior rather than absolute figures, sales is distributed on the range $[0,1]$, that is $x \sim U[0,1]$, throughout the analysis. Table 2 provides a numerical example and a comprehensive comparison between the different strategies.

7.1 Influence of Online Store's Return Rate

Online returns negatively impact all parties involved, as the increase in the online store's return rate leads to a decline in overall performance and individual participants' performance due to the associated burden. However, the system and its players demonstrate greater immunity with the presence of re-salability. This is evident from the relatively higher and less steep decline in both system profitability and players' performance as the return rate r_o increases (Figure 1a-c).



(a) (b) (c) (d) (e) (f)

Figure 1 Influence of online store's return rate ($r_o = variable$, $r_r = 0.15$, $p = 200$, $h_r = 20$, $h_l = 5$, $w = 40$, $s = 10$, $g = 5$, $\emptyset = 0.5$, $\theta = 0.2$).

Radhi (2018) has demonstrated that the provider consistently achieves higher performance under a gain-sharing compensation policy compared to a flat-fee compensation policy. Meanwhile, the retailer must account for market conditions (e.g., return rate, pricing, etc.) and market power (e.g., market share) to determine the optimal partnership strategy. Our findings confirm that this remains true even when re-salability is considered (Figure 1a). Literature confirms that partnering with a third-party logistics provider does not always lead to improved performance for the retailer. Instead, a well-calibrated and carefully considered decision is essential (Hartmann & de Grahl, 2012). However, it is important to note that the complexity of decision-making and potential disagreements in crafting an agreement can be mitigated through a full gain-sharing arrangement. This approach is typically supported by the existence of a minimum acceptable share for both parties, making full centralization more advantageous than other coordination schemes (Figure 1f). However, an exception arises when the provider is exceptionally powerful, with values of $\emptyset$ approaching zero. In such cases, no minimum acceptable share can satisfy both parties, disrupting the centralization process. Surprisingly, these shares are not significantly affected by the presence of re-salability.

With re-salability in mind, the retailer can intuitively lower the quantity level under almost any strategy or market conditions, though some exceptions exist, which will be addressed later (Figure 1e). Conversely, the provider may view re-salability as a challenge, as it demands greater logistical effort, leading to higher fixed-fees for each managed online item, where applicable (Figure 2d). Consistent with existing literature, the retailer should reduce the online store's order quantity to mitigate the impact of returns, which, in turn, prompts the provider to lower the fixed-fee.

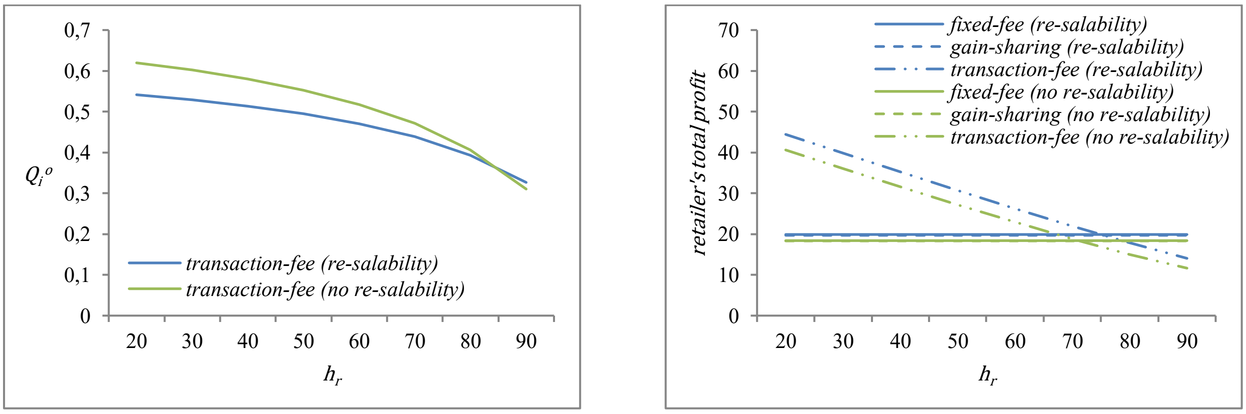
A key factor contributing to high return rates for online stores is the lenient return policy. Such a policy is implemented to reassure customers about product quality, as online shoppers cannot physically inspect items before purchasing (J. Liu *et al.*, 2020; Vlachos & Dekker, 2003). Hence, fashion, accessories, and jewelry often have higher return rates because customers frequently order multiple sizes or styles, returning those that do not meet their preferences. Similarly, furniture returns are common due to unclear representations of size, fit, or style online. The need for assembly also increases return rates, as customers may find setup challenging.

Moreover, the accessibility of online products has extended their selling seasons compared to physical store products. Available 24/7 to a global audience, online products allow customers to shop anytime, often sustaining demand even after traditional seasons end. For instance, seasonal items like holiday decorations or winter coats continue selling online to customers in different regions or climates.

This combination of high return rates and extended selling seasons has increased the potential for return re-salability. As a result, retailers and providers are encouraged to revisit their decisions, optimizing both individual and overall profitability.

7.2 Influence of Retailer's Logistical Cost

Under a transaction-based fee strategy, it is notable that at high values of h_r, an online store's order quantity in a system with re-salability surprisingly exceeds that of a system without re-salability (Figure 2a). This is because non-re-salable units generate lower profits with higher logistical fees, leading to a sharp decline in order quantities. In contrast, re-salable units demonstrate greater resilience to higher logistical costs, serving as a buffer to mitigate this drop. This highlights the importance for retailers to not assume that order quantities are always lower in re-salable systems. Furthermore, as the retailer's logistical cost increases, a system with non-re-salable products requires earlier involvement of a provider compared to a system with re-salable products (Figure 2b).



(a) (b)

Figure 2 Influence of retailer's logistical cost ($r_o = 0.2$, $r_r = 0.15$, $p = 200$, $h_r = variable$, $h_l = 5$, $w = 40$, $s = 10$, $g = 5$, $\emptyset = 0.5$, $\theta = 0.2$).

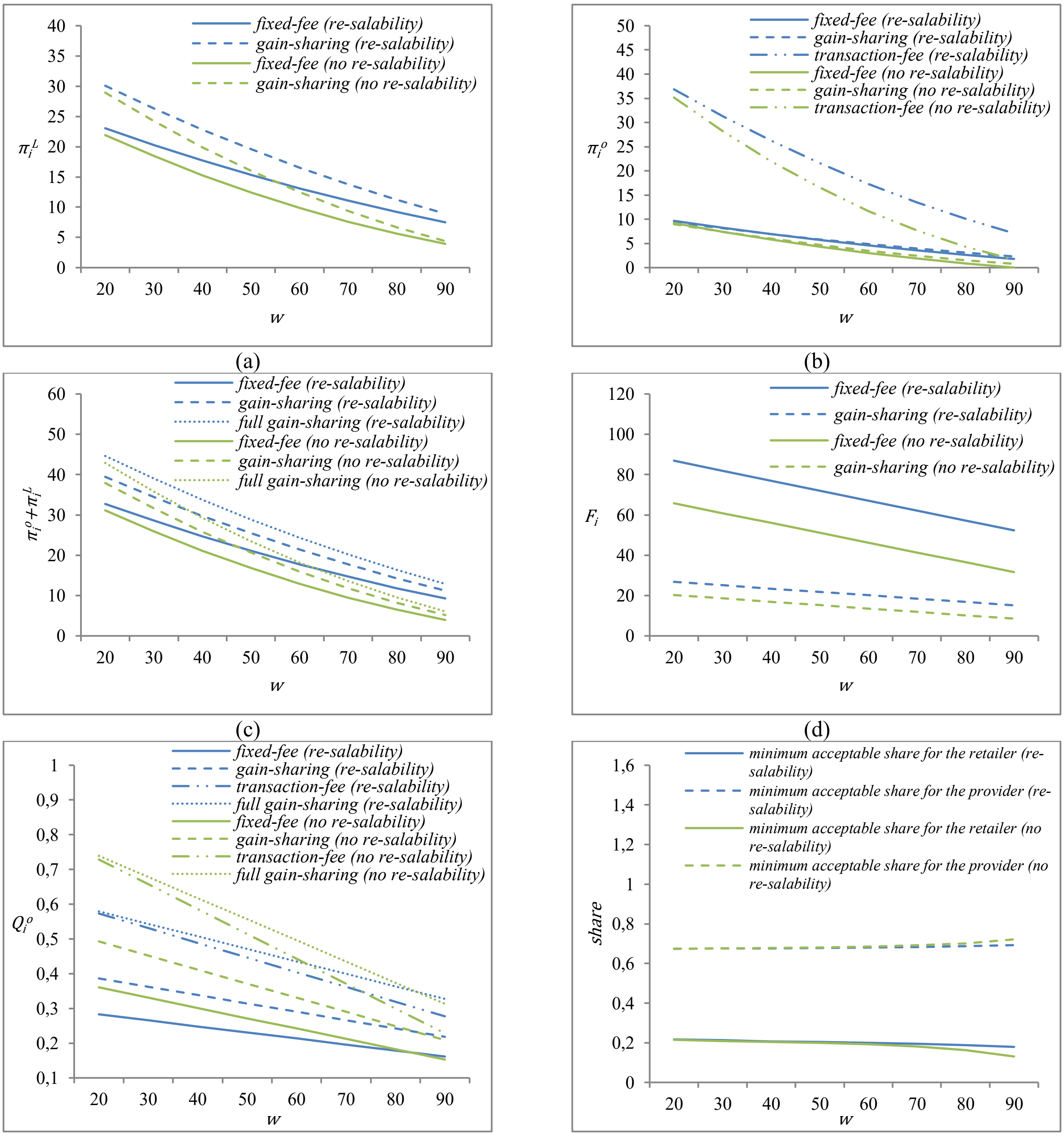
In practice, retailers often face higher logistical costs due to several factors. One significant reason is the lack of economies of scale. Unlike logistics providers, retailers typically handle smaller shipping volumes, limiting their ability to benefit from bulk transportation discounts. This makes handling, delivering, and returning items more expensive. Additionally, retailers often exhibit inefficiencies in warehousing and transportation management because their primary focus is on selling products rather than optimizing logistical operations. Furthermore, many retailers operate both physical and online stores from premium locations, which contribute to higher logistics-related operational costs. In contrast, logistics providers tend to operate warehouses and fulfillment centers in less expensive areas strategically chosen for transportation efficiency rather than customer accessibility. Such factors may necessitate a reassessment of the retailer's optimal decisions and strategic approaches.

7.3 Influence of Unit Purchasing Cost

As the unit purchasing cost rises, the burden on all parties increases, prompting the retailer to lower inventory levels regardless of the strategy employed or the presence of re-salability (Figure 3e). This reduction in inventory compels the provider to decrease the fixed-fee to help the online store maintain its market share (Figure 3d). If the provider fails to offer support, the retailer's optimal strategy would involve further inventory reduction, potentially causing significant market share losses.

In systems with re-salability, the online store's order quantity tends to exceed that of non-re-salable systems at higher purchasing costs (Figure 3e). Re-salable systems are better equipped to stay active in the market, whereas non-re-salable systems show a tendency to retreat as costs rise. This underscores the importance for retailers to avoid assuming that order quantities are always lower in re-salable systems.

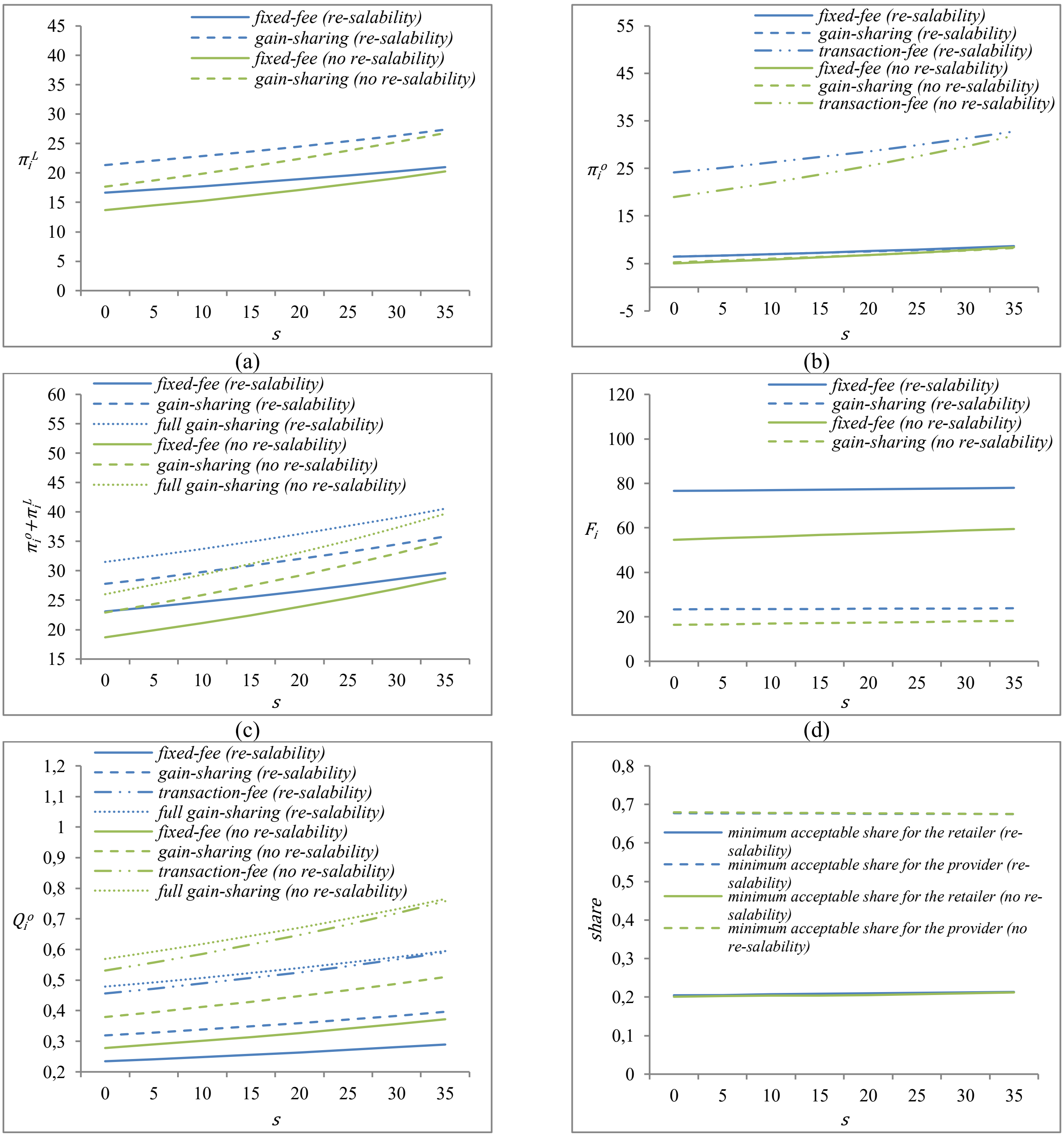
High purchasing costs naturally diminish the profitability of channels and partners. However, systems with re-salability demonstrate greater resilience and an enhanced ability to withstand crises (Figure 3a-c). As such, it is essential for partners to explore strategies to enhance re-salability, particularly in the context of rising costs. In addition, non-re-salable systems exhibit divergence in the provider's minimum acceptable shares at high values of w (Figure 3f). That is, an unusual deviation from the typical scenario where provider's and retailer's minimum acceptable shares remain consistent and unaffected by re-salability.



(a) (b) (c) (d) (e) (f)

Figure 3 Influence of unit purchasing cost ($r_o = 0.3, r_r = 0.15, p = 200, h_r = 20, h_l = 5, w = variable, s = 10, g = 5, \emptyset = 0.5, \theta = 0.2$).

In the previous analysis, it was observed that an increase in the unit purchasing or production cost, while the selling price remains constant, places significant strain on profit margins. This phenomenon can arise in various scenarios. For instance, the cost of raw materials may experience a sudden increase due to natural disasters. Similarly, labor costs may escalate as a result of labor strikes or other workforce-related challenges. Additionally, energy and shipping costs can surge due to political conflicts, such as the Russian-Ukrainian crisis. Such conflicts have had divergent impacts on global competitiveness, benefiting certain countries while disadvantaging others. Geopolitical factors, including tariffs and trade restrictions imposed on specific countries or materials, further contribute to the increase in purchasing costs. These factors, among others, can lead to a rise in unit costs while the selling price remains unchanged due to the availability of comparable products competing within the same market. Therefore, it is crucial to coordinate optimal decisions with unit purchasing or production costs to achieve improved performance.



(a) (b)
(c) (d)
(e) (f)

Figure 4 Influence of salvage value ($r_o = 0.3$, $r_r = 0.15$, $p = 200$, $h_r = 20$, $h_l = 5$, $w = 40$, $s = 10$, $g = variable$, $\emptyset = 0.5$, $\theta = 0.2$).

7.4 Influence of Salvage Value

An increase in salvage value reduces the need to impose re-salability on a retailing system. Retailers handling non-re-salable returns would experience less financial strain with higher salvage values (Figure 4a-c). This alleviates the performance pressure and consequently increases their order quantities (Figure 4e). However, retailers dealing with re-salable returns are less influenced by salvage values, leading to only a slight increase in their order quantities. Moreover, although the retailer's performance varies significantly under a full gain-sharing strategy compared to a transaction-fee strategy, online inventory levels converge to nearly equivalent values as the parameter s increases. It is also noted that providers' performance improves in such cases, whereas their decisions remain largely unchanged (Figure 4d).

In addition, when salvage values approach the unit purchasing or production costs, both parties become almost indifferent to whether the returns are re-salable or non-re-salable (Figure 4a-c). In fact, retailers become indifferent to the coordination strategy used and instead move toward a more independent state (Figure 4b). Finally, changes in salvage values fall within the typical scenario where the provider's and retailer's minimum acceptable shares remain consistent, regardless of re-salability (Figure 4f).

In practice, salvage value can be increased through several effective channels. Retailers can partner with specialized liquidators, such as B-Stock and Tech Liquidators, which focus on reselling excess inventory in bulk. Discount stores or outlets like TJ Maxx and Best Buy Outlet are also potential channels for selling such items. Additionally, retailers can explore secondary national or international markets where customers are willing to pay a premium. For instance, companies often distribute reliable and affordable excess products to emerging markets, as seen in Nokia's success in major parts of Africa and India. Modern platforms like eBay and Amazon also provide significant opportunities to sell surplus inventory directly to a broad audience.

However, it is crucial to approach these channels at the right time and in the right location. For instance, companies can sell surplus summer apparel through channels that target customers living in year-round hot climates. By identifying such channels, retailers can focus their efforts on salvage process rather than investing excessive resources into analyzing re-salability or coordination strategies.

8. CONCLUSION

Contemporary retail enterprises increasingly adopt a dual-channel strategy, integrating both physical and online stores to cater to the diverse purchasing preferences of customers. A significant number of these dual-channel retailers offer full refunds, which has led to a substantial increase in customer returns, particularly within the online channel. Thus, considering return re-salability is crucial for retailers, as it directly impacts inventory management, profitability, and sustainability by enabling the reintegration of returned items into the supply chain, thereby reducing waste and optimizing resource allocation. Furthermore, in light of ongoing supply chain disruptions and global economic challenges, many dual-channel retailers are focusing on their core business activities while outsourcing the logistical operations of their online stores to third-party logistics and service providers.

This study re-investigates three contractual agreements that dual-channel retailers may consider for managing the online store. The first agreement is the transaction-fee model, wherein the retailer either utilizes its private fleet for all logistical operations or compensates a provider on a per-delivery basis. The second agreement is the fixed-fee model, in which the retailer delegates all logistical responsibilities for the online store to a provider, paying a fixed-fee for each item processed. The third agreement is the gain-sharing model, which fosters a stronger partnership, allowing the provider to receive a percentage of the online store's gain alongside a per-item fixed-fee. For each strategy, a profit maximization model is developed to determine the optimal inventory levels for both channels and, where applicable, the optimal fixed fee, while accounting for the re-salability of product returns. This framework facilitates a comparative analysis between two retail strategies: one where a dual-channel retailer restocks returned items for resale, and another where a retailer salvages returns upon receipt. To enable a tractable analysis, customer demand is assumed to follow a uniform distribution.

While the provider perceives re-salability as a logistical challenge, requiring greater effort to manage each online item, the retailer views it as an opportunity to reduce waste and enhance system efficiency. Consequently, the provider tends to impose higher fixed-fees, while the retailer reduces order quantities under nearly all strategies or market conditions. However, certain exceptions arise, highlighting the need for retailers to avoid assuming that order quantities are always lower in re-salable systems.

For instance, under a transaction-based fee strategy, when the retailer's logistical costs are high, an online store may order more in a re-salable system than in a non-re-salable one. This is because non-re-salable units yield lower profits under high logistical fees, leading to a sharp decline in order quantities. In contrast, re-salable units are more resilient to rising logistical costs, acting as a buffer against this reduction. A similar trend occurs across all studied strategies when purchasing costs are high, as re-salable systems are more adaptable and better positioned to remain competitive, while non-re-salable systems tend to scale back as costs increase.

High purchasing costs and customer returns reduce the profitability of channels and partners. However, systems with re-salability show greater resilience and are better able to withstand burdens. Therefore, it is crucial for partners to explore strategies that strengthen their partnership and enhance re-salability, especially in the face of rising costs and increasing customer returns. In contrast, high salvage values enhance the profitability of channels and partnerships but diminish the necessity of implementing re-salability within a retailing system. For instance, when salvage values approach the unit purchasing or production costs, both parties show little preference for whether returns are re-salable or not. In such cases, retailers become largely indifferent to the coordination strategy employed and tend to operate in a more independent manner.

Lastly, it is worth emphasizing that the complexity of decision-making and potential conflicts in reaching an agreement can be alleviated through a full gain-sharing arrangement. This approach is generally favored due to the existence of a minimum acceptable share for both parties, making full centralization more beneficial than other coordination methods. However, an exception occurs when the provider holds disproportionate power. In such scenarios, no minimum acceptable share can satisfy both parties, hindering the centralization process. Interestingly, the presence of re-salability has little impact on these shares.

This paper has several limitations related to model setting and assumptions. First, the study examines an interaction between a weaker, follower retailer and a stronger, leader provider. Future research could explore retailer-provider coordination in scenarios where the retailer has greater bargaining power, either through simultaneous or sequential games. Second, a more realistic model would allow retailers to set channel prices using a differential pricing strategy, providing greater flexibility in negotiating terms. Third, with advancements in logistics and evolving customer purchasing behavior, a more realistic assumption regarding re-salability should be adopted—specifically, repetitive re-salability. That is, an item can be returned and resold multiple times, potentially an infinite number of times within a selling season, until it is either salvaged or retained by a customer. While this assumption has yet to gain widespread acceptance in the scholarly community, it warrants reconsideration.

REFERENCES

Akçay, Y., Boyacbox, T., & Zhang, D. (2013). Selling with money-back guarantees: The impact on prices, quantities, and retail profitability. *Production and Operations Management*, *22*(4), pp. 777–791. https://doi.org/10.1111/j.1937-5956.2012.01394.x

Balakrishnan, A., Sundaresan, S., & Zhang, B. (2014). Browse-and-switch: Retail-online competition under value Uncertainty. *Production and Operations Management*, *23*(7), pp. 1129–1145. https://doi.org/10.1111/poms.12165

Berger, P. D., Lee, J., & Weinberg, B. D. (2006). Optimal cooperative advertising integration strategy for organizations adding a direct online channel. *Journal of the Operational Research Society*, *57*(8), pp. 920–927. https://doi.org/10.1057/palgrave.jors.2602069

Cai, X., Chen, J., Xiao, Y., Xu, X., & Yu, G. (2013). Fresh-product supply chain management with logistics outsourcing. *Omega (United Kingdom)*, *41*(4), pp. 752–765. https://doi.org/10.1016/j.omega.2012.09.004

Cao, K., Xu, Y., Hua, Y., & Choi, T. M. (2023). Supplier or co-optor: Optimal channel and logistics selection problems on retail platforms. *European Journal of Operational Research*, *311*(3), pp. 971–988. https://doi.org/10.1016/J.EJOR.2023.06.005

Chen, J., & Bell, P. C. (2012). Implementing market segmentation using full-refund and no-refund customer returns policies in a dual-channel supply chain structure. *International Journal of Production Economics*, *136*(1), pp. 56–66. https://doi.org/10.1016/j.ijpe.2011.09.009

Chen, J., & Grewal, R. (2013). Competing in a supply chain via full-refund and no-refund customer returns policies. *International Journal of Production Economics*, *146*(1), pp. 246–258. https://doi.org/10.1016/j.ijpe.2013.07.005

Chen, J., & Zhang, H. (2011). The impact of customer returns on competing chains. In *International Journal of Management Science and Engineering Management* (Vol. 6, Issue 1). http://www.ijmsem.org/

Chen, X. X., Xu, P., Li, J. J., Walker, T., & Yang, G. Q. (2022). DECISION-MAKING IN A RETAILER-LED CLOSED-LOOP SUPPLY CHAIN INVOLVING A THIRD-PARTY LOGISTICS PROVIDER. *Journal of Industrial and Management Optimization*, *18*(2), pp. 1161–1183. https://doi.org/10.3934/jimo.2021014

Chiang, W. Y. K., & Monahan, G. E. (2005). Managing inventories in a two-echelon dual-channel supply chain. *European Journal of Operational Research*, *162*(2), pp. 325–341. https://doi.org/10.1016/j.ejor.2003.08.062

Fabbe-Costes, N., Jahre, M., & Roussat, C. (2009). Supply chain integration: The role of logistics service providers. *International Journal of Productivity and Performance Management*, *58*(1), pp. 71–91. https://doi.org/10.1108/17410400910921092

Fallahi, N., Hafezalkotob, A., Raissi, S., & Ghezavati, V. (2023). Cooperation and coopetition among retailers-third party logistics providers alliances under different risk behaviors, uncertainty demand and environmental considerations. *Environment, Development and Sustainability*, *25*(6), pp. 5597–5633. https://doi.org/10.1007/s10668-022-02282-x

Giri, B. C., & Sarker, B. R. (2017). Improving performance by coordinating a supply chain with third party logistics outsourcing under production disruption. *Computers and Industrial Engineering*, *103*, pp. 168–177. https://doi.org/10.1016/j.cie.2016.11.022

Goertler, T., Papert, M., & Fischer, I. (2024). Theorizing Supply Chain Management and Logistics Research: A Comprehensive Review of Trends, Gaps, and Future Directions. *OPERATIONS AND SUPPLY CHAIN MANAGEMENT*, *17*(4), pp. 329–343.

Hartmann, E., & de Grahl, A. (2012). Logistics outsourcing interfaces: the role of customer partnering behavior. *International Journal of Physical Distribution & Logistics Management*, *42*(6), pp. 526–543. https://doi.org/10.1108/09600031211250578

He, L.-H., Liao, X., Zhang, G., & Haug, P. (2016). Analysis of online dual channel supply chain based on service level of logistics and national advertising. *Quality Technology & Quantitative Management*, *13*(4), pp. 473–490. https://doi.org/10.1080/16843703.2016.1191181

Jharkharia, S., & Shankar, R. (2007). Selection of logistics service provider: An analytic network process (ANP) approach. *Omega*, *35*(3), pp. 274–289. https://doi.org/10.1016/j.omega.2005.06.005

Jiang, L., Wang, Y., & Yan, X. (2014). Decision and coordination in a competing retail channel involving a third-party logistics provider. *Computers and Industrial Engineering*, *76*(1), pp. 109–121. https://doi.org/10.1016/j.cie.2014.07.026

Jin, D., Caliskan-Demirag, O., Chen, F. (Youhua), & Huang, M. (2020). Omnichannel retailers' return policy strategies in the presence of competition. *International Journal of Production Economics*, *225*. https://doi.org/10.1016/j.ijpe.2019.107595

Keränen, J., Kienzler, M., Salonen, A., Terho, H., & Totzek, D. (2023). Gain-sharing in performance-based contracting: How risk and fairness drive business customers' willingness-to-switch to a gain-sharing arrangement. *Industrial Marketing Management*, *115*, pp. 172–184. https://doi.org/10.1016/j.indmarman.2023.09.013

Kim, S., & Lee, C. (2024). Study Examines Role of Collaboration-Enhancing Factors in Supply Chain. *OPERATIONS AND SUPPLY CHAIN MANAGEMENT*, *17*(1), pp. 77–88.

Lei, L., Wang, Q., & Fan, C. (2006). Optimal business policies for a supplier–transporter–buyer channel with a price-sensitive demand. *Journal of the Operational Research Society*, *57*(3), pp. 281–289. https://doi.org/10.1057/palgrave.jors.2602000

Liinamaa, J., Viljanen, M., Hurmerinta, A., Ivanova-Gongne, M., Luotola, H., & Gustafsson, M. (2016). *Performance-based and functional contracting in value-based solution selling*. https://doi.org/10.1016/j.indmarman.2016.05.032

Lim, W. S. (2000). *Theory and Methodology A lemons market? An incentive scheme to induce truth-telling in third party logistics providers*.

Liu, J., Ren, M., Yang, A., & Feng, S. (2020). Money-Back Guarantee, Dual Money-Back Guarantee, and O2O Strategy in a Manufacturer's Dual-Channel Supply Chain. *Mathematical Problems in Engineering*. https://doi.org/10.1155/2020/1697082

Liu, W. H., Xu, X. C., & Kouhpaenejad, A. (2013). Deterministic approach to the fairest revenue-sharing coefficient in logistics service supply chain under the stochastic demand condition.

Computers and Industrial Engineering, *66*(1), pp. 41–52. https://doi.org/10.1016/j.cie.2013.06.008

Min, H. (2013). Examining logistics outsourcing practices in the United States: From the perspectives of third-party logistics service users. *Logistics Research*, *6*(4), pp. 133–144. https://doi.org/10.1007/s12159-013-0110-8

Mostard, J., De Koster, R., & Teunter, R. (2005). The distribution-free newsboy problem with resalable returns. *International Journal of Production Economics*, *97*(3), pp. 329–342. https://doi.org/10.1016/j.ijpe.2004.09.003

Mostard, J., & Teunter, R. (2006). The newsboy problem with resalable returns: A single period model and case study. *European Journal of Operational Research*, *169*(1), pp. 81–96. https://doi.org/10.1016/j.ejor.2004.04.048

Nageswaran, L., Cho, S. H., & Scheller-Wolf, A. (2020). Consumer return policies in omnichannel operations. *Management Science*, *66*(12), pp. 5558–5575. https://doi.org/10.1287/mnsc.2019.3492

Ofek, E., Katona, Z., & Sarvary, M. (2011). "Bricks and clicks": The impact of product returns on the strategies of multichannel retailers. *Marketing Science*, *30*(1), pp. 42–60. https://doi.org/10.1287/mksc.1100.0588

Prasertwit, T., Kanchanasuntorn, K., & Vongmanee, V. (2024). Green E-commerce Supply Chain Management. *OPERATIONS AND SUPPLY CHAIN MANAGEMENT*, *17*(3), pp. 141–151.

Radhi, M. (2022). Service Decisions in a Two-Echelon Retailing System with Customer Returns. *Journal of Theoretical and Applied Electronic Commerce Research*, *17*(3), pp. 1219–1242. https://doi.org/10.3390/jtaer17030062

Radhi, M. (2024). MANAGING THE ONLINE CHANNEL BY COORDINATING A THIRD-PARTY LOGISTICS AND SERVICE PROVIDER ALONG WITH A DUAL-CHANNEL RETAILER. *International Journal of Industrial Engineering : Theory Applications and Practice*, *31*(3), pp. 577–598. https://doi.org/10.23055/ijietap.2024.31.3.9813

Radhi, M. H. (2018). *Optimization of a Dual-Channel Retailing System with Customer Returns*. University of Windsor.

Radhi, M., & Zhang, G. (2018). Pricing policies for a dual-channel retailer with cross-channel returns. *Computers and Industrial Engineering*, *119*. https://doi.org/10.1016/j.cie.2018.03.020

Radhi, M., & Zhang, G. (2019). Optimal cross-channel return policy in dual-channel retailing systems. *International Journal of Production Economics*, *210*, pp. 184–198. https://doi.org/10.1016/j.ijpe.2019.01.014

Reimann, M. (2016). *Accurate Response with Refurbished Consumer Returns * Subject Areas: Accurate Response, Consumer Returns, Refurbishing, and Closed-loop Supply Chain* (Vol. 47). https://doi.org/10.1111/(ISSN)1540-5915

Ryan, J. K., Sun, D., & Zhao, X. (2013). Coordinating a supply chain with a manufacturer-owned online channel: A dual channel model under price competition. *IEEE Transactions on Engineering Management*, *60*(2), pp. 247–259. https://doi.org/10.1109/TEM.2012.2207903

Tu, J., Sun, Z., & Huang, M. (2022). Supply chain coordination considering e-tailer's promotion effort and logistics provider's service effort. *Journal of Industrial and Management Optimization*, *18*(3), p. 2191. https://doi.org/10.3934/jimo.2021062

Vlachos, D., & Dekker, R. (2003). Return handling options and order quantities for single period products. *European Journal of Operational Research*, *151*(1), pp. 38–52. https://doi.org/10.1016/S0377-2217(02)00596-9

Wang, K.-H., Tung, C.-T., Lee, Y.-J., Wang, K. H., Tung, C. T., & Lee, Y. J. (2010). EOQ model for deteriorating items with resalable returns. In *Journal of Information & Optimization Sciences* (Vol. 31, Issue 1). http://www.rt-mart.com.tw/corp4.asp

Wang, Y., Yu, Z., Shen, L., Fan, R., & Tang, R. (2021). Decisions and coordination in e-commerce supply chain under logistics outsourcing and altruistic preferences. *Mathematics*, *9*(3), pp. 1–23. https://doi.org/10.3390/math9030253

Yan, R. (2008). Pricing strategy for companies with mixed online and traditional retailing distribution markets. *Journal of Product and Brand Management*, *17*(1), pp. 48–56. https://doi.org/10.1108/10610420810856512

Yao, D. Q., Yue, X., Mukhopadhyay, S. K., & Wang, Z. (2009). Strategic inventory deployment for retail and e-tail stores. *Omega*, *37*(3), pp. 646–658. https://doi.org/10.1016/j.omega.2008.04.001

Yu, M. C., & Goh, M. (2012). Time-bound product returns and optimal order quantities for mass merchandisers. *International Journal of Systems Science*, *43*(1), pp. 163–179. https://doi.org/10.1080/00207721.2010.481367

Zhang, A., Ren, J., Guan, Z., & Farooq, U. (2021). Decision and Coordination in the Dual-Channel Supply Chain Considering the Risk-Averse and Customer Returns. *Journal of Mathematical Finance*, *11*(01), pp. 48–83. https://doi.org/10.4236/jmf.2021.111003

Zhang, J., Farris, P. W., Irvin, J. W., Kushwaha, T., Steenburgh, T. J., & Weitz, B. A. (2010). Crafting integrated multichannel retailing strategies. *Journal of Interactive Marketing*, *24*(2), pp. 168–180. https://doi.org/10.1016/j.intmar.2010.02.002

Zhang, S., Mei, Y., Bao, Q., & Shao, L. (2021). Coordination Strategy for a New Retail Supply Chain Based on Combination Contract. *Complexity*, *2021*. https://doi.org/10.1155/2021/7697429

Dr. Mohannad Radhi is an assistant professor of Industrial Engineering at King Abdulaziz University. He earned his Ph.D. in Industrial and Manufacturing Systems Engineering from the University of Windsor, Ontario, Canada, in 2018. His research focuses on decision-making in the areas of remanufacturing, quality control, dual-channel supply chains, customer returns, inventory control, pricing policies, and service management. His work has been published in several highly reputable journals, including the International Journal of Production Economics, International Journal of Production Research, Computers & Industrial Engineering, among others.