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# Shelf-space Allocation Model with Demand Learning

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## ABSTRACT

**In this paper, we studied the shelf-space allocation problem (SSAP). It is quite common recently to implement product design during a selling season and drastically change assortment decisions based on shelf-space allocation in response to up-to-date demand observations. While there are many literatures related to SSAP, However, existing literature assume that the demand is stationary. In this paper, we propose a dynamical framework to make shelf-space display decisions, in which space elasticity and potential demand are sequentially estimated using the latest data containing display space and sales for each product.**

**Keywords**: *shelf-space allocation problem, retail operations, demand management*

## 1. INTRODUCTION

Internet technology has developed rapidly in the past few decades. Owning a smartphone or PC has now become commonplace. Thus, e-commerce has also been expanding rapidly, especially in the retail industry. However, the business-to-consumer e-commerce market in product sales only covers only a few percent of the total product sales in many countries; in other words, store sales still account for a large proportion of the total sales. Therefore, in-store merchandizing (ISM) is still important to store operation.

Retailers have limited shelf space available. Thus, some of the critical issues for category managers to maximize profitability of each category are (1) which products to include in an assortment; (2) how to allocate these products to shelves. The first issue is related to *assortment planning*, and the second issue is related to *shelf-space planning*. With an increasing availability of marketing data on which to base decisions, the solution of these problems via mathematical optimization techniques has been drawing increased attention.

In this study, we study the shelf-space allocation problem (SSAP). A key assumption in SSAP is the *space elasticity*. The space elasticity is the relationship between product sales and the number of product facings. This relation is mostly modeled as a concave function. Existing research based on this relationship. SSAP is well studied problem and many papers has proposed models to incorporate a variety of factors, such as inventory integration and cross-elasticity. Existing papers, however, assumed only stationary demand such as daily commodities. Stores that deal with seasonal or brief cycle products (e.g., apparel retailer) revise the store design and display design every week via empirical determination. Existing models are not applicable to these real situations because nonstationary demand had not been considered. Therefore, this paper proposes a method for deciding the weekly display space for each product following nonstationary demand. Normally, using the sales data, retail stores can check the demand for each product. Thus, space elasticity and potential demand are sequentially estimated using the latest data that contains the display space and sales for each product. By taking nonstationary demand into account, the potential demand is estimated for each week, and then using estimated formula, the optimal display space for the following week is decided on an ongoing basis. In our numerical experiment, the proposed method was compared with the conventional model executed sequentially, using the calculated objective function: profit for the whole period. In conclusion, this paper proposes a simulation model to make a decision regarding the optimal weekly display space following nonstationary demand and allows retail stores to increase total profit. The proposed model is also useful when the store deals with seasonal or brief cycle products and frequently revises its display design.

## 2. LITERATURE REVIEW

The SSAP is a well-studied problem. See Karampatsa *et al*. (2017) for an extensive review.

It is well known that the sales of an item are correlated with its shelf space, or specifically the number of product facings (e.g., Curhan (1972) and Eisend (2014), the relationship of which is modeled as a concave function. This correlation is known as *space elasticity*, and most of the proposed models attempt to maximize profits while taking this feature into account. One of the earliest examples of research in this area was by Hansen and Heinsbroek and Heinssbroek (1979), who proposed a model and an algorithm for the simultaneous optimal selection among a given set of products from the assortment of products to be sold in a supermarket and the allocation of shelf space to these products.

The trend toward more comprehensive decision support models is based on two research streams. In the first, there is a growing acceptance among practitioners and corporations of the necessity for an integrated view of the shelf-space allocation and inventory decision. Urban (1998) proposed a model to integrate existing inventory-control models, product assortment models, and shelf-space allocation models. Hwang et al. (2005) developed an integrated mathematical model for the SSAP and inventory-control problem with the objective of maximizing the retailer's profit, and proposed a gradient search heuristic and a genetic algorithm for the solution to the model. Hariga *et al*. (2007) proposed a joint optimization model for inventory replenishment, product assortment, and shelf-space and display-area allocation decisions. Abbott and Palekar (2008) studied a single-store multi-product inventory problem in which product sales are a composite function of shelf space considering demand depletion. Ramaseshan *et al*. (2008) studied a model to maximize the total net profit in terms of decision variables expressing product assortment, shelf-space allocation, review period, and order quantity. Hubner and Kuhn (2011a, 2011b) proposed a retail shelf-space management model with space-elastic demand and consumer-driven out-of-assortment substitution effects.

In the other stream, there has been expansion to integrate the model with own-space and cross-space elastic demand. Cross-elasticity means the responsiveness of the demand for one product to a change in the price of another product. In this case, the number of facings of one product has an effect on the demand for another product. Corstjens and Doyle (1981) proposed a model for optimizing retail-space allocations to maximize store profitability expressed by the demand function, where both main and cross-space elasticities have to be considered, and through the cost function (procurement, carrying, and out-of-stock costs). Bultez *et al*. (1989) proposed a model to consider asymmetric cannibalism in retail assortments and proposed heuristics to solve the problem. Borin *et al*. (1994) developed a category management model to aid retailers in the space-constrained decisions of which products to stock (assortment) and how much shelf space to allocate to those products, and proposed a simulated-annealing-based heuristic. Lim *et al*. (2004) extended the model to address other requirements such as product groupings and nonlinear profit functions, and proposed an algorithm to combine local search with a metaheuristic approach.

There are several other marketing effects to take into consideration. As mentioned above, because SSAP is highly related to the sales and profit of a store, there is also research that considers customer incentives driven by promotions (Tsao *et al*., 2014) and customer service factors (Reyes and Frazier (2007)). Yang and Chen (1999) proposed a model to include the location effect in which item location has a major impact on its sales as well as the number of facings. Murray *et al*. (2010) developed a model that jointly optimizes a retailer's decisions for product prices, display facing areas, display orientations, and shelf-space locations in a product category, taking account of multiple product orientations that capture three-dimensional product packaging characteristics. Bai *et al*. (2008) added a location effect to the proposed model.

As SSAP is complex nonlinear problem that involves many realistic factors as mentioned above, the solution procedure become computationally challenging. Consequently, some models are intractable with off-the-shelf solvers, and thus several efficient algorithms to solve the problem have been proposed. Zufryden (1986) proposed a dynamic programming approach for product selection and supermarket shelf-space allocation. Yang (2001) proposed heuristics motivated by multi-constraint knapsack problems. Hansen *et al*. (2010) proposed a linear programming formulation for a retail shelf-space decision model that incorporates a nonlinear profit function, vertical and horizontal location effects, and product cross-elasticity, and proposed heuristic-based and meta-heuristics-based algorithms that are much faster than the standard linear programming solver. Gajjar and Adil (2010) and Irion *et al*. (2010) proposed a piecewise linearization technique for approximating the complicated nonlinear model of SSAP, in which inventory level and cost were added. Gajjar and Adil (2010) proposed local search heuristics to solve the retail SSAP with a linear profit function, which creates an initial arrangement and then uses adjustment moves to iteratively improve the profit of the current solution. An efficient simulated-annealing multiple-neighborhood hyper-heuristic approach was proposed for this two-dimensional SSAP.

Even with progress, however, none of cited literature considers demand learning, and accordingly the SSAPs are static, rather than explicit dynamic models. Typical retailers, such as fashion retailers, have both items that are in demand in certain quantities constantly over time and items that can be in demand for a limited selling season. With the increasing availability of data today, it is very important to determine demand constantly and respond to changes in customer needs quickly. In this paper, we propose a model to maximize the effect of a more aggressive response to dynamic demand.

# 3. PROBLEM FORMULATION

In space allocation optimization, space elasticity is often considered. Although product sales depend on the number of product facings, the relationship is not a linear, but a concave function as shown in Figure 1. Thus, the retailer should display not only a well-selling product but also several types of products. In general, space elasticity is estimated by simple methods such as ordinary least squares regression or more complex methods such as multi-stage least squares and seemingly unrelated regression.

In initial research focused on space elasticity, Curhan (1972) tried to estimate the space elasticity from 11 product characteristics by using multiple regression analysis. However, the coefficient of determination was .032 and this regression analysis has little power to predict the space elasticity. In addition, Curhan observed nearly 500 grocery products under actual operating conditions, and the average space elasticity was .212 for all items, which shows a positive relationship between shelf space and unit sales. Eisend (2014) surveyed 31 conventional studies that were published from 1960 to summer 2012 and these reported 1,268 space elasticities. Eisend observed that the mean shelf-space elasticity was 169.

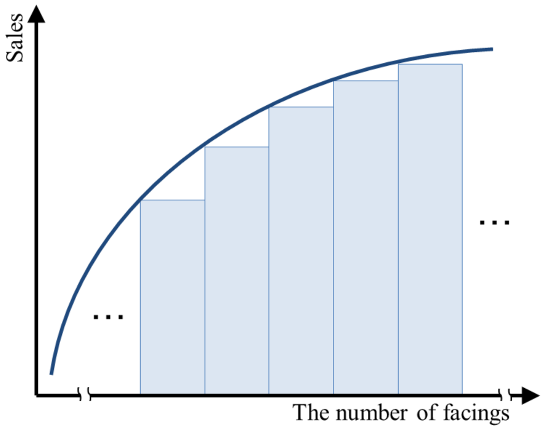


**Figure 1.** Relationship between the number of facings and sales for one product

The SSAP is representative of shelf-space management and this has an important influence on the product sales. In the first step of a purchasing decision process, large amounts of information are acquired by the sense of sight, especially when considering impulse buying. Thus, if a product is given a large shelf space, it is more likely to be seen by customers in a store and this increases the opportunity to be bought. This problem is often formulated as a resource allocation problem for the purpose of maximizing profits. Taking space elasticity into account, this problem becomes a nonlinear problem.

A standard SSAP can be formulated as follows:

$$\text{Maximize } \sum_{i=1}^{n} p_i \alpha_i x_i^{\beta_i} \quad (1)$$

$$\text{Subject to } \sum_{i=1}^{n} a_i x_{ik} \le T_k, \quad k = 1, \cdots, m \quad (2)$$

$$x_i = \sum_{k=1}^{m} a_i x_{ik}, \quad i = 1, \cdots, n \quad (3)$$

$$L_i \le x_i \le U_i, \quad i = 1, \cdots, n \quad (4)$$

$$x_{ik} \in \mathbb{Z}^+, \quad i = 1, \cdots, n, \quad k = 1, \cdots, m \quad (5)$$

$$x_i \in \mathbb{Z}^+, \quad i = 1, \cdots, n \quad (6)$$

- $p_i$ is the unit profit of product $i$
- $D_i$ is the demand for product $i$;
- $x_i$ is the number of facings of product $i$ on all shelves;
- $x_{ik}$ is the number of facings of product $i$ on shelf $k$;
- $\alpha_i$ is the scaling constant of product $i$;
- $\beta_i$ is the space elasticity of product $i$;
- $a_i$ is the face length of product $i$;
- $T_k$ is the length of shelf $k$;
- $L_i$ is the minimum number of facings of product $i$;
- $U_i$ is the maximum number of facings of product $i$.

Equation (1) means maximizes retailer's total profit. Constraint (2) ensures that total length of products does not exceed the given shelf length. Constraint (3) defines that $x_i$ is equal to the sum of $x_{ik}$ on all shelves. Constraint (4) ensures that each the total number of facings has both lower limit and upper limit. Constraints (5) and (6) mean the number of facings is a non-negative integer.

We consider the problem in a dynamic setting. The retailer should revise the store design every week using previous sales data and other data, if products have large fluctuations in demand. The proposed simulation model optimizes display space every period using the latest sales and display space data. Then, it obtains sales data as feedback about the decision made for this period. In the next period, it repeats this process using the latest data, including the decision and feedback from the previous period. The proposed simulation method is described as follows.

**Step 1** : Set $t = 1$
**Step 2** : Set $i = 1$
**Step 3** : Estimate $\alpha_{it}$ and $\beta_{it}$ of category $i$ in period $t$ from historic data.
**Step 4** : If $i < I$ then Set $i \coloneqq i + 1$ and go to step 3, otherwise go to step 5.
**Step 5** : Solve display space in period $t$
**Step 6** : Run sales simulation in period $t$
**Step 7** : If $t < T$ then Set $i \coloneqq i + 1$ and go to step 2, otherwise end.

Although conventional research assumed only stationary demand, nothing has been proposed to determine space allocation following nonstationary demand. This paper changes the existing simplified model into a dynamic model and proposes a simulation model to make the decision regarding the weekly optimal display space following nonstationary demand. Previously, a scale parameter α was used as a constant; however, in this paper we see the scale parameter $\alpha$ as a function of period $t$. In fact, each period has a different value of $\alpha$, thus $\alpha$ cannot be estimated as a constant by regression analysis calculation. This paper proposes to calculate $\alpha$ every period and predicts time series by using fluctuations in $\alpha$. Therefore, this paper compares three methods to predict $\alpha$: estimation by conventional regression analysis in each period; prediction by weighted moving average; and prediction by difference sequence. Each method is described in Section 3.4.

Our model to optimize display space can be formulated as follows:

$$\text{Maximize} \quad \sum_{t=1}^{T} \sum_{i=1}^{I} p_i \alpha_{it} \left[\frac{s_{it}}{s_i}\right]^{\beta_{it}} \quad (7)$$

$$\text{Subject to } \sum_{i=1}^{I} S_{it} \le C \qquad t = 1, \cdots, T \tag{8}$$
$$L_i \le \left\lceil \frac{S_{it}}{s_i} \right\rceil \le U_i \quad i = 1, \cdots, n, t = 1, \cdots, T \tag{9}$$

- $p_i$ is the unit profit of category $i$;
- $D_{it}$ is the demand for category $i$ in period $t$;
- $\alpha_{it}$ is the scaling constant of category $i$ in period $t$;
- $\beta_{it}$ is the space elasticity of category $i$ in period $t$;
- $S_{it}$ is the face length of category $i$ in period $t$;
- $s_i$ is the unit space of category $i$ in period $t$;
- $C$ is the capacity of display space in the store;
- $L_i$ is the minimum number of facings of category $i$;
- $U_i$ is the maximum number of facings of category;

# 4. PROPOSED METHOD

We proposed three different methods to incorporate the dynamic feature.

First method is to estimate $\beta_{it}$ using the display space and sales data of each category from period $t - n$ to $t - 1$, $\alpha_{it}$ and. In this model, demand is formulated as a nonlinear function using

$$D_{it} = \alpha_{\text{it}} \left\lceil \frac{S_{it}}{s_i} \right\rceil^{\beta_{it}} = \alpha_{it} x_{it}^{\beta_{it}} \tag{10}$$

Taking the logarithm of both sides and adding an error term, we have

$$\log(D_{it}) = \log(\alpha_{it}) + \beta_{it} \log(x_{it}) L + \epsilon_{it}$$
$$\epsilon_{\text{it}} \sim N(0, \sigma_{it}^2) \tag{11}$$

Equation (11) is a logarithmically transformed version of Equation (10) and is a linear function. Therefore, $\alpha_{it}$ and $\beta_{it}$ can be estimated by simple linear regression analysis and logarithmic transformation. This is a static method. However, it can cover a dynamic situation if it is implemented every period.

Second method is to estimate $\alpha_{it}$ using the display space and sales data of each category from period $t - n$ to $t - 1$. First, $\alpha_{i(t-n)}, \cdots, \alpha_{i(t-1)}$ must be calculated by using the following equation and weights $w_{i(t-n)}, \cdots, w_{i(t-1)}$ have to be set:

$$\alpha_{it} = \frac{D_{it}}{\left\lceil \frac{S_{it}}{s_i} \right\rceil^{\beta_{it}}} \tag{12}$$

Then, $\alpha_{it}$ are predicted using

$$\alpha_{it} = \sum_{k=1}^{n} w_{i-k} \alpha_{i(t-k)} + \epsilon_{it}$$
$$\epsilon_{\text{it}} \sim N(0, \sigma_{it}^2) \tag{13}$$

Third method is to estimate $\alpha_{it}$ using the display space and sales data of each category from period $t - n$ to $t - 1$. First, $\alpha_{i(t-n)}, \cdots, \alpha_{i(t-1)}$ must be calculated using Equation (12). In this model, we use the lag operator $\Delta^d$ as follows:

$$\Delta^d \alpha_{it} = \Delta^{d-1} \alpha_{it} - \Delta^{d-1} \alpha_{i(t-1)} \tag{14}$$

For examples with $d = 1,2$, Equation (15) shows the differences in each term of sequence $\alpha_i$ and Equation (16) shows the differences each term of sequence $\Delta\alpha_i$:

$$\Delta\alpha_{it} = \alpha_{it} - \alpha_{i(t-1)} \tag{15}$$
$$\Delta^2\alpha_{it} = \Delta\alpha_{it} - \Delta\alpha_{i(t-1)} \tag{16}$$
$$= \alpha_{\text{it}} - 2\alpha_{i(t-1)} + \alpha_{i(t-2)}$$

Then, $\alpha_{it}$ are predicted using the following equation, where $d$ denotes the difference degree used for making the time series stationary:

$$\alpha_{it} = \sum_{t=1}^{d-1} \Delta^l \alpha_{i(t-1)} + \Delta^d \hat{\alpha}_{it} + \epsilon_{it}$$
$$\epsilon_{\text{it}} \sim N(0, \sigma_{it}^2) \tag{17}$$

This model is the same in spirit as an autoregressive integrated moving average model, taking the difference in the time series $\alpha_{it}$ to render it stationary. It is particularly advantageous when the demand time series is nonstationary.

# 5. NUMERICAL EXPERIMENT

Our experiment assumes a store that deals with categories that have both stationary demand and seasonal demand. The capacity of display space in the store ($C$) is 50 m$^2$ and the assortment of products has 10 categories (4 stationary categories and 6 seasonal categories). The assortment of products has already been decided by the retailer. Each category must be displayed in at least one unit ($L_i$) and at most 40% of capacity ($U_i$) because the retailer wishes to sell a variety of products. When the retailer determines the display space, they refer to only the latest five periods of sales and display space data ($n = 5$). The input data of $\alpha$ and $\beta$ are referred only by the sales simulation. Thus, optimizing the display space uses estimated or predicted $\alpha$ and $\beta$. Figure 2 shows fluctuations of the input data $\alpha$. Table 1 lists the other input data.

In the numerical experiment, three methods were conducted and the sales and mean absolute percentage error (MAPE) were compared. The results of the simulation for each method is given in the following (Table 2).

From Table 2, Method 1 could not estimate the parameter. In other words, simple linear regression analysis could not follow demand fluctuation. Thus, it could not optimize the display space and resulted in the lowest sales. Method 2 produced a better sales performance, however it was a few periods behind in following demand fluctuation. Weighted moving average is a smoothing method; thus, it could not respond to demand fluctuation immediately. Method 3 performed the lowest MAPE and produced the best sales performance. By using a difference sequence, it could follow demand fluctuation immediately, thus it could optimize the display space and produced a better sales performance. Figure 3 shows the display space for each period using Method 3.

In this numerical experiment, sales simulation had an error term as other fluctuating factors. The result above has some variability: $\epsilon \sim N(0,3^2)$. If other fluctuating factors have a greater effect on the sales, variance in the error term becomes larger. Therefore, additional experiments were performed. Table 3 lists the results of the simulation where the variance of the error term was equal to $5^2$ and Table 4 lists the results where the variance was equal to $7^2$.

In both cases, Method 3 produced the best result and Method 1 produced the lowest sales and highest MAPE in each case. Conventional research on space elasticity or space allocation was based on estimation by linear regression analysis. However, taking large demand fluctuations into account, linear regression analysis is not suitable because it needs data from several periods that include fluctuation in demand. In this situation, predicting time series using fluctuation in $\alpha$ is better than estimating $\alpha$ and $\beta$ by linear regression analysis.

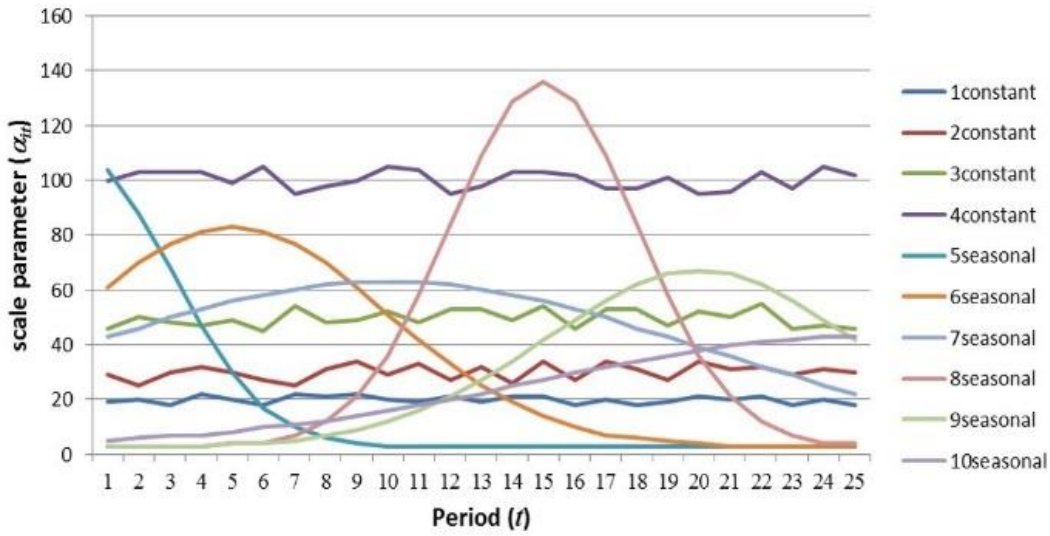


**Figure 2** Fluctuation of the scale parameter

**Table 1** Fluctuation of the scale parameter

| # | Seasonality | $\beta_i$ | unit sales (JPY) | unit pace (m²) |
|---|---|---|---|---|
| 1 | Stationary | 0.17 | 800 | 0.15 |
| 2 | Stationary | 0.15 | 1,200 | 0.15 |
| 3 | Stationary | 0.20 | 2,500 | 0.20 |
| 4 | Stationary | 0.18 | 1,000 | 0.25 |
| 5 | Seasonal | 0.23 | 1,800 | 0.25 |
| 6 | Seasonal | 0.28 | 2,000 | 0.30 |
| 7 | Seasonal | 0.35 | 3,000 | 0.35 |
| 8 | Seasonal | 0.32 | 1,500 | 0.20 |
| 9 | Seasonal | 0.22 | 3,500 | 0.30 |
| 10 | Seasonal | 0.29 | 5,000 | 0.35 |

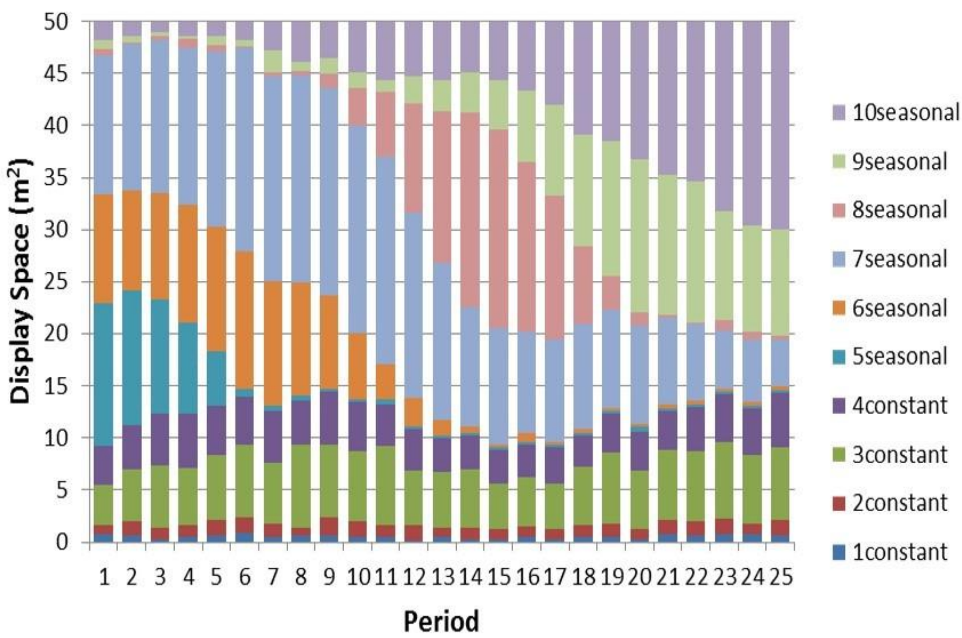


**Figure 3** Display space for each period (Method 3)

**Table 2** Result of Simulation: $\epsilon \sim N(0,3^2)$

| | Method 1 | Method 2 | Method 3 |
|---|---|---|---|
| **Actual Sales (JPY)** | 34,411,300 | 40,553,300 | 41,023,500 |
| **Estimated Sales (JPY)** | 37,192,248 | 40,971,310 | 41,181,402 |
| **MAPE** | 12.32% | 7.58% | 2.05% |

**Table 3** Result of Simulation: $\epsilon \sim N(0,5^2)$

| | Method 1 | Method 2 | Method 3 |
|---|---|---|---|
| **Actual Sales (JPY)** | 34,081,800 | 40,704,000 | 41,152,400 |
| **Estimated Sales (JPY)** | 37,149,253 | 41,147,058 | 41,800,370 |
| **MAPE** | 12.25% | 8.07% | 4.85% |

**Table 4** Result of Simulation: $\epsilon \sim N(0,7^2)$

| | Method 1 | Method 2 | Method 3 |
|---|---|---|---|
| **Actual Sales (JPY)** | 37,288,300 | 40,773,500 | 41,125,400 |
| **Estimated Sales (JPY)** | 44,128,834 | 41,244,288 | 41,944,979 |
| **MAPE** | 22.87% | 8.45% | 6.71% |

# 6. CONCLUSION

In this paper, we have proposed a simulation model to determine the weekly optimal display space following demand fluctuation. Using a difference sequence produced the best sales and most accurate estimation of the three proposed methods. Conventional research on shelf-space allocation tends to devise a creative solution to the optimization approach, and the scale parameter $(\alpha)$ and space elasticity $(\beta)$ are given in these models. In contrast, our approach takes demand fluctuation into account and predicts parameters dynamically and then optimizes the display space for each period. In this case, we observed that conventional linear regression analysis is not suitable. In the past, a retail store that deals with seasonal or brief cycle products may have revised store design by empirical determination. However, our method can provide an optimal determination and contributes to increasing sales.

For future research, we should also consider other forecasting method such as the state-space based model. We should consider an integrated model with assortment planning, inventory replenishment. We should also consider optimization model to incorporate the error of model. This can be considered with the stochastic programming approach.

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